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APPLIED MECHANICS REVIEWS

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RECENT DEVELOPMENTS IN THE STUDY OF BUCKLING PROBLEMS

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THE study of buckling problems began in 1844 with Euler's famous column theory. Up until comparatively recent times theoretical studies in this field were confined, like Euler's, to the idealized case of the neutral equilibrium for infinitesimal displacements of a perfectly elastic structure having a shape and loading for which such a condition exists. By the 1930's reasonably exact solutions of this classical stability theory had been obtained for most of the more basic cases, such as straight and circular rods, rectangular plates and circular cylinders, with clamped, hinged, and free boundaries, loaded to produce uniform compression or shear parallel or perpendicular to the boundaries.

The engineers who had to use such elements in their machines and structures early found that these "exact solutions" sometimes showed little relationship to actual behavior. Thus the classical theory is satisfactory for very slender struts, but because of the limits to the elastic behavior of actual materials, empirical results are relied on to this day for most applications. When classical stability theories became available for more complex elements, it was found that nonelastic behavior was only one of the causes for serious discrepancies between such theories and experiments. For example, the classical theory predicts many times the actual strengths of very thin shells under axial compression; on the other hand, the classical theory predicts only a fraction of the actual ultimate strengths of very thin hinged or fixed edge plates under compression or shear. Since these discrepancies become greater the thinner the material and hence the smaller the average stress, they can hardly be due to nonelastic effects.

Adequate theories are needed even more for the more complex cases than for the case of simple struts, because the greater number of variables makes it less practical to cover all combinations by empirical methods. The rise of the aeronautics industry, with its need for weight efficiency, has necessitated more realistic studies of many buckling problems, with a consideration of not only the factors considered in the classical theory but of many other factors which sometimes have an important effect upon the behavior of actual specimens—nonelastic as well as elastic behavior of materials, finite displacements, initial imperfections which are always present, inertia, etc. There is an obvious parallel in this with the history of fluid mechanics—the early preoccupation of theoreticians with idealized "perfect fluids," the failure to answer many practical questions and the consequent reliance upon empirical methods, until aeronautical problems forced the consideration of important additional factors affecting real phenomena.

CLASSICAL STABILITY STUDIES

The above introduction does not mean, of course, that classical stability studies are useless or out of date. In a great many cases the effects of large displacements and of inertia, etc., are unimportant, the structure is not particularly sensitive to initial imperfections, and stresses are within the elastic range until the peak resistance is reached, and so the classical analysis is quite adequate. Even in cases where it is not adequate it is the obvious first approximation and steppingstone for further, more complex studies, or a basis for semi-empirical design methods.

But it is very important to remember that this is a field where pitfalls await the unwary, and where experimental confirmation is very essential, especially when the investigator is on unfamiliar ground or when classical stability theory has been found wanting in similar cases. Even experimental confirmation in a narrow range may be misleading when complicating factors have opposing tendencies. For example, experiments have sometimes given fortuitous checks with the simple classical analysis of a curved panel under axial compression, but only because one factor—the same as for flat panels—increases the strength several times, while another factor—the same as for complete cylinders under axial compression—decreases it several times! Tests of supposedly hinged end struts are sometimes reported as giving strengths above the Euler value; this is not an indication of a good test but suggests rather that considerable resistance to end rotation must have been present, the resulting partial end fixity more than balancing the effects of imperfections present.

The following reviews refer to some of the interesting classical studies which have appeared in recent years: Buckling of bars: AMR 1, Revs. 256, 1620; 4, Rev. 1090. Buckling of simple plates and shells: AMR 1, Revs. 437, 439, 441; 2, Revs. 596, 727, 1125, 1254; 3 Rev. 2263; 4, Revs. 143, 1533, 2380, 4121, 4421; 5, Rev. 1355. There has been a large output of classical studies of anisotropic and composite plates and shells. Sandwich materials are covered in: AMR 1, Rev. 65; 3, Revs. 680, 1470, 1673, 2624; 4, Revs. 160, 646, 1123, 1997, 2437, 3840, 4122, 4124; 5, Rev. 1040. Plywood construction: AMR 1, Revs. 258, 1327, 1328, 1330, 1459; 2, Revs. 25, 33, 34, 36; 3, Revs. 679, 867, 2258; 4, Rev. 1099. Among many classical studies of stiffened

plates and shells are: AMR 1, Revs. 67, 623, 810, 951, 1326, 1616; 2, Rev. 851; 3, Revs. 865, 869, 870; 4, Revs. 2424, 2425, 4422. Classical studies of the buckling of structures made up of a number of connected elements are obviously important but have received less attention: AMR 1, Revs. 621, 1619; 2, Revs. 604, 857; 3, Rev. 2633; 4, Revs. 174, 2832; 5, Revs. 1037, 1372.

EFFECT OF NONELASTIC MATERIAL BEHAVIOR

In all cases of buckling, nonelastic behavior of the material occurs at some time during the buckling process. If it occurs after the peak resistance has been reached, its effect is not likely to be of much practical importance. But if nonelastic behavior occurs before the peak resistance is reached, then the details of this behavior determine the magnitude of the peak resistance and are of great importance. For materials which yield at a constant stress, the average buckling stress cannot greatly exceed this yield stress unless the proportions are very stocky; however, this is only a rough rule, and tells little about constructions made of other materials.

The onset of buckling of a perfect specimen for which a condition of neutral equilibrium is possible, is mathematically a singular point in the load-displacement relation. This causes no great difficulty when the material behaves elastically. However, when the complication is added of a singularity in the stress-strain relation of yielding material (which follows one law for increasing, and another for decreasing stresses), the solution and interpretation of the theoretical relations become quite confusing and obscure, even for the comparatively simple case of a strut. This matter has received considerable discussion and clarification in recent years: AMR 1, Rev. 72; 3, Revs. 50, 1079; 4, Revs. 1047, 1091, 1522, 1525, 1987. However, it might be pointed out that to some extent this confusion is academic and unnecessary. Due to ever present initial imperfections or other disturbing factors, true neutral equilibrium is impossible in actual specimens. The consideration of such factors is difficult but not impossible. Taking them into consideration would eliminate one of the foregoing singularities and with it the confusions associated with nonelastic buckling, and would, moreover, provide a more realistic analysis.

The case of the nonelastic buckling of plates and shells has received much attention in spite of its still greater complications. Recent work on this problem has been reviewed in a recent survey article [AMR 4, page 587].

EFFECTS OF FINITE DEFORMATIONS AND INITIAL IMPERFECTIONS

Classical bar, plate, and shell bending theories approximate curvatures by the second derivative of lateral displacements, and in the case of plates and shells neglect "large deflection" membrane strains and stresses. The error made in the expression for curvature is seldom of practical importance, but it has been found necessary to take large deflection membrane stresses into account to explain actual buckling behavior in certain important cases. One of these cases is that of the ultimate strength of compressed panels. While a remarkably good approximate solution was obtained by von Kármán more than twenty years ago, using small deflection theory and semi-intuitive reasoning, it has long been recognized that more complete analysis requires large-deflection theory. Recent work in this field includes: AMR 1, Rev. 1196; 2, Revs. 994, 995; 3, Revs. 49, 1668, 1671, 1889, 2259.

By far the most striking example of inadequacy of small deflection theory is the case of axial compression of thin cylinders or curved sheet. Von Kármán and Tsien showed that post-buckling behavior in this case is vastly different from that predicted by "small deflection" theory, not only when the deflections are large but right from the beginning when the deflections are small. Recent work in this direction includes: AMR 2, Revs. 1255, 1377; 4, Rev. 2859. While large deflection studies of perfect specimens demonstrated the great sensitivity of this case to disturbances or imperfections, it was necessary to take these factors into consideration in the theory in order to explain completely the relatively low peak buckling resistance (using the word "buckling" in a broad sense) of actual specimens [AMR 4, Rev. 635; 5, Rev. 1341]. Recent unpublished studies show that the same assumptions regarding initial imperfections which explain the relatively low peak resistance of axial compression cyclinders also explain the relatively high resistance of torsion buckling specimens.

EFFECTS OF INERTIA

A number of studies have recently been made of the effects of inertia on buckling under very rapidly varying axial loads: AMR 2, Rev. 1120; 3, Revs. 861, 1892; 4, Revs. 1096, 1136, 1990, 2434; 5, Revs. 76, 1036.

Theoretical and Experimental Methods

(See also Revs. 1945, 1961, 1980, 2106, 2127, 2147, 2175, 2196)

1923. Shinbrot, M., A description and a comparison of certain nonlinear curve-fitting techniques, with applications to the analysis of transient-response data, *NACA TN* 2622, 41 pp., Feb. 1952.

Let $q(t)$ be a quantity which theoretically should satisfy an identity of the form

$$q(t) = e^{\alpha t}(A \cos \omega t + B \sin \omega t)$$

Suppose experimental values of $q(t)$ are measured at a series of points $t_0, t_1, t_2, \ldots, t_n$. The problem specifically studied is that of finding the best values of the constants α, ω, A, and B in the "least squares" sense.

A description and critical comparison are given of a number of applicable curve-fitting techniques including several steepest descent methods, relaxation method, and Taylor series method. The different methods are applied to a numerical example using actual flight data obtained by measuring the pitching velocity of a test airplane in response to an elevator pulse. It is pointed out that the methods discussed apply to a wide variety of curve-fitting problems, including ones which involve more complicated functional relations than that of the specific example.

Dana Young, USA

1924. Matthieu, P., Error estimation for method of Adams (differential equations). I. Equations of first order (in German), *ZAMM* **31**, 11/12, 356–370, Nov./Dec. 1951.

Paper deals with the first-order differential equation $y' = f(x,y)$. Let $y(x)$ with $y(x_0) = y_0$ be a solution and $z(x)$ with $z(x_0) = y_0$ an approximation for $y(x)$. Then the error $d(x) = y(x) - z(x)$ and the functions $p(x) = -\partial f(x,z)/\partial y$, $q(x) = f(x,z) - z'$ are introduced. The error satisfies approximately $d' + pd = q$. This equation is used for estimating d in the extrapolation method of Adams; here $q(x)$ is replaced by a function $\bar{q}(x)$ with greater absolute value, $\bar{q}$ being a saw-tooth function with linear shape in each integration interval. A routine of estimating d is developed. A numerical example for $f(x,y)$ both linear in x and y shows that the estimation does not exceed a factor 1.5 as compared with the real error.

Hans Bückner, Germany

1925. Courbon, J., Numerical integration in strength of materials (in French), *Ann. Inst. Bât. Trav. publics* (N.S.) no. 240, 11 pp., Feb. 1952.

This is a clear exposition of many of the conventional methods for the numerical evaluation of integrals. The presentation is

made entirely from the viewpoint of the engineer using these methods, and is free of any obscuring details. About a dozen formulas are listed in the logical order in which they arise from Newton's interpolation polynomial, together with estimates of the errors with which they are affected. As is done so often, the first omitted term of a Taylor expansion is accepted as a measure of the error. After a good discussion which shows that each integration problem needs to be examined carefully to determine which integration formula is most suitable for it, author comes out, somewhat illogically, with a recommendation for one particular procedure (Simpson's rule followed by a certain third-order formula) to be followed in all kinds of mechanical load problems. There is a commendable discussion of the (often disregarded) danger in applying numerical quadrature formulas if the integrand is not continuous and continuously differentiable. It might have been added that discontinuities in the second or higher derivatives are equally dangerous. Paper concludes with three numerical examples taken from structural mechanics. Appended to the paper is a transcript of the discussion which followed its oral presentation, including a brief exposition of graphical integration methods by R. Vallette.

Franz L. Alt, USA

1926. **Rutishauser, H., Speiser, A., and Stiefel, E., Sequence controlled digital computers (electronic computers)** (in German), *ZAMP* **1**, 2, 5, 6, 1, 2: 277–296, 339–362, 1–24, 63–92; Sept., Nov. 1950, Jan., Mar. 1951.

This is a relatively complete (94 pages) expository treatment of automatic digital computers both of the United States and other countries. Article covers types of problems most suitable for automatic computation, and describes the structure of such computing machines. It contains a partial list of computing machines either completed or under construction in 1949) with their characteristics. Arithmetic principles, such as the binary and decimal systems, and the fixed binary or decimal point method vs. floating point operation, are discussed. Logical details of computers are considered, and the use of flow diagrams and other coding details are explained.

The use of relays and of vacuum-tube circuits to accomplish the logical operations is covered by the authors, and various types of memory storage including acoustic delay lines, magnetic drums, and electrostatic are described. Harry D. Huskey, USA

1927. **Bubb, F. W., Jr., A circuit for generating polynomials and finding their zeros,** *Proc. IRE* **39**, 12, 1556–1561, Dec. 1951.

Paper describes an instrument for generating polynomials and for finding the roots, real or complex, of any polynomial, of any degree, with either real or complex coefficients, and to any degree of accuracy. It is also shown that this instrument may be built with commercially available parts.

From author's summary

1928. **Aparo, E., and Dainelli, D., The EDSAC, a modern electronic computer** (in Italian), *Ric. sci.* **22**, 2, 186–201, Feb. 1952.

The report starts with a description of the main aspects of the electronic computer EDSAC. Some examples follow, with the aim of showing how the mathematician has to give the machine the necessary information in order that it may perform the calculations and write the results. From authors' summary

1929. **Langefors, B., Improvement in electric computer networks for some elastic structures,** SAAB Aircr. Co., Linköping TN 1, 16 pp., 1951.

Author suggests method of setting up electric network analogs for solution of equilibrium states of certain mechanical phenomena whereby only a single parameter of the analog must be varied to accommodate the variation of a single mechanical parameter.

Method is restricted to those problems in which the mechanical parameter to be varied may be factored out of the transformation matrix representing the set of equations defining stationary state of the phenomenon, and in which the elements of the transformation matrix satisfy the relation $a_{ij}a_{nn} = a_{in}a_{nj}$, $[i, j = 1, 2, \ldots\ldots (n-1)]$.

Method, in brief, consists of adding an admittance branch and a junction point to a network so that new admittance A_n has the value $A_n = A_{nn}/(1 - 1/k)$. Here n represents number of the node to which admittance is connected, and k is factor by which the mechanical parameter is varied. The electrical analog is still considered to have original number of nodes with new node substituted for the one to which additional admittance is connected.

A further practical restriction which author states is $0.25 \leq k \leq 4$.

Specific examples given by author are pin-jointed beams in two- and three-dimensional space, and a "shear plate" with two infinitely rigid edge stiffeners. In these examples, parameter which is varied is stiffness.

It is suggested that a set of standard electrical networks can be stockpiled and an analog set up for a specific problem with only minor alterations. Leonard Becker, USA

1930. **Herr, D., Two new economical computers for the design and analysis of servomechanisms, networks, amplifiers, and analogous dynamical systems,** *J. Amer. Soc. nav. Engrs.* **63**, 4, 851–871, Nov. 1951.

Analog computing techniques are applied to two problems: (1) To evaluate polynomials and rational functions of a complex variable over any domain of the variable; (2) to determine the locus of roots of expressions of the form

$$(1 + a_1z)\ldots(1 + a_nz) - K(1 + b_1z)\ldots(1 + b_mz) = 0$$

for any range of values of K or the a's and b's. The latter parameters may be complex quantities. The relation of these problems to servomechanism analysis is discussed. Author presents block diagrams of the computers. Components include servos, a-c induction potentiometers [title source, **63**, 1, Feb. 1951], and a-c resolvers (rotatable transformers), for which author claims accuracy of 0.05%. E. Bromberg, USA

1931. **Collatz, L., On the stability of the difference method in rod vibrations** (in German), *ZAMM* **31**, 11/12, 392–393, Nov./Dec. 1951.

The partial differential equation $\{\partial^4/\partial x^4 + K\partial^2/\partial t^2\}\, U = 0$ for a vibrating rod is replaced by an infinite system of difference equations for points distributed in a place-time coordinate-plane (rectangular meshes). Mash width in x and t direction is respectively, l and h. The difference equation for $U_{j,k+1}$ (approximating value to $u(x_j, t_{k+1})$ is

$$U_{j,k+1} = -U_{j,k-1} - zU_{j+2,k} + 4zU_{j+1,k} + (2 - 6z)U_{j,k} + 4zU_{j-1,k} - zU_{j-2,k}(z = h^2/Kl^4)$$

$U_{j,k+1}$ is calculated from the right-hand side of this equation. Author investigates how z must be chosen in order that small errors in the values $U_{j,k}$ and $U_{j,k+1}$ for a certain k will not lead to an unbounded error distribution in the values $U_{j,k}$ for $k \to \infty$. Author shows that for a rod on two supports with $u(x, 0) = f_1(x)$ and $\partial u(x, 0)/\partial t = f_2(x)$, it is sufficient that $z \leq 1/4$.

J. W. Cohen, Holland

1932. Petrovskii, I. G., Lectures on the theory of integral equations [Lektsii po teorii integral'nikh uravnenii] 2d ed., Moscow-Leningrad, Gosud. Izd. Tekh.-Teor. Lit., 1951, 127 pp.

Author gives an exposition of the fundamental theorems of integral equations of the second kind

$$\varphi(P) = \int_G K(P, Q)\varphi(Q)dQ + f(P)$$

where $P, Q\epsilon$, and G are a d-dimensional region bounded by a finite number of $(d - 1)$-dimensional smooth surfaces. In chapter 1, the Fredholm theorems are stated and then proved, first for a degenerate kernel, next for continuous kernels small in absolute value, then for uniformly continuous kernels, and finally for kernels of the form $\bar{K}(P, Q)/PQ^\alpha$ where $\alpha < d$ and $\bar{K}$ is uniformly continuous. In chapter 2, Volterra equations are treated briefly by reduction to the last case of chapter 1. In chapter 3, real symmetric piecewise continuous kernels are treated. Here the author emphasizes the analogy with transformations of n-dimensional vectors by symmetric matrices, setting forth the analogous theorems in parallel columns. The rest of the chapter is then devoted to proving these theorems for the integral equation case. In an appendix, the author indicates the necessary modifications in order to carry through the theorems of chapter 3 for real symmetric Lebesgue square integrable kernels. The book is written simply and clearly and with good motivation for the theorems.

J. V. Wehausen, USA

1933. Esipovich, E. M., On stability of solutions of a class of differential equations with retarded argument (in Russian), *Prikl. Mat. Mekh.* **15**, 601–608, Sept./Oct. 1951.

Differential-difference systems of the form

$$dy(t)/dt = Ay(t) + By(t - h)$$

lead to transcendental equations of the form

$$P(z) + Q(z)e^{-z} = 0$$

where P and Q are polynomials, for the determination of the characteristic roots. The author presents some general results on the location of the roots of this equation and treats some particular equations by way of illustration.

R. Bellman, USA

1934. Shulgin, M. F., Reduction of a system of differential equations to the Lagrange form (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **75**, 3, 349–351, Nov. 1950.

Given the system

$$\ddot{q}_r = f_r(t;\ q_i;\ \dot{q}_i)\ (i, r = 1, \ldots, n) \qquad [1]$$

introduce the supplementary coordinates q_{n+i} and put

$$L = \Sigma_i \dot{q}_i \dot{q}_{n+i} + \Sigma_i f_i q_{n+i}$$

Then the first n Lagrange equations derived from L define the q_{n+i}, while the last n coincide with the original system [1]. If there are $k(k < n)$ cyclic coordinates, the Lagrangian system of order $2n$ can be reduced to a new Lagrangian system of order $2n - k$, the new coordinates being $q_{k+1}, \ldots, q_{2n}$, and the new kinetic potential $R = L - \Sigma_{i=1}^{K}(\partial L/\partial \dot{q}_i)\dot{q}_i$ containing k arbitrary constants c_r. When the latter system has been solved, the remaining original coordinates q_r can be obtained from the equation

$$q_r = \int \frac{\partial R}{\partial c_r} dt + b_r,\ (r = 1, \ldots, k) \qquad [2]$$

where b_r are constants of integration.

If the coordinates $q_i[i = 1, \ldots, n)$ are not explicitly contained in the f_r, they are also not contained in L. In this special case of $k = n$ cyclic coordinates, a particular solution of the reduced Lagrangian system for the q_{n+i} yields, by means of the equation [2], the general solution of the original system.

E. Leimanis, Canada

1935. Bonneau, E., and Janin, R., Calculation of complex roots of algebraic equations of higher degree with real or complex coefficients and mechanization of the method on punched-card machines (in French), *Rech. aéro.* no. 25, 39–54, Jan.-Feb. 1952.

Authors describe, in detail, Graeffe's method for the solution of an algebraic equation $P_0(z) = 0$ (with real coefficients). To compute the real parts of two conjugate complex roots and the sign of the real roots, the equation $P_0(z - \epsilon) = 0$ is treated equally, i.e., the roots are displaced by the very little distance ϵ along the real axis. The method is extended to equations with complex coefficients, and prepared for the use of punched-card machines. The reckoning time is given for some examples that were used as test equations; e.g., the solution of an equation of the 10th degree with real coefficients required 2 hr 40 min, if all roots were real, twice the time if some roots were complex, and eight times the time in the case of complex coefficients.

F. A. Willers, Germany

1936. Prodi, G., Stability questions for nonlinear partial differential equations of parabolic type (in Italian), *Atti Accad. naz. Lincei R. C. Sci. Fis. Mat. Nat.* (8) **10**, 365–370, 1951.

The problem of demonstrating the stability in the sense of Poincaré-Lyapunov of the solution $u = 0$ of the nonlinear parabolic equation $u_t = u_{xx} + u_{yy} + u_{zz} + f(x, y, z, t, u)$, where $u = a(x, y, z)$ at $t = 0$ for $(x, y, z)\epsilon R$, $u = 0$ on B, the boundary of R, and $|f|/|u| \to 0$ as $u \to 0$, was raised by the reviewer [*Trans. Amer. math. Soc.* **64**, 21–44, 1948] and solved for some simple regions, in particular for a rectangular parallelepiped. The author, using some very simple methods tracing back to Haar and Westphal, proves the stability for general bounded regions and for more general forms of the parabolic equation. He further shows that the condition $|f|/|u| \to 0$ may be replaced by the weaker one $|f(u_1) - f(u_2)| \leqq c_1|u_1 - u_2|$, where c_1 is a constant which depends upon R. Although the question of the existence of the solution of the above equation is not discussed, the methods used in the paper are equally applicable to the treatment of this problem.

R. Bellman, USA

1937. Prodi, G., New stability criteria for the equation $y'' + A(x)y = 0$. I (in Italian), *Atti Accad. naz. Lincei R. C. Sci. Fis. Mat. Nat.* (8) **10**, 447–451, 1951.

It was shown by Ascoli that all solutions of (1) $u'' + (1 + a(t))u = 0$ are bounded if $a(t)$ is the sum of a function of bounded variation over $[t_0, \infty]$ and a function which is absolutely integrable over $[t_0, \infty]$. Subsequently, it was shown by the same author that if $a(t) = t^{-1} \sin \alpha t$, a function which escapes the previous criteria, then the solutions are bounded if $\alpha \neq 2$, and unbounded otherwise. In the present note is shown that the solutions of $u'' + (1 + t^{-6} \sin \alpha t)u = 0$ are bounded if $\alpha \neq 2$ and $6 > \frac{1}{2}$. This is a special case of a general result obtained by converting (1) into an integral equation $u = v + {}_0\int^t k(t, t_1)a(t_1)u(t_1)dt_1$, where v is the general solution of $u'' + u = 0$, and iterating once, obtaining

$$u = v + {}_0\int^t k(t, t_1)a(t_1)v(t_1)dt_1 + {}_0\int^t k(t, t_1)a(t_1) \left({}_0\int^{t_1} k(t_1 t_2)a(t_2)k(t_2)dt_2\right) dt_1.$$

R. Bellman, USA

1938. **Bulgakov, B. V., The discriminant curve and the domain of aperiodic stability** (in Russian), *Prikl. Mat. Mekh.* **14**, 453–458, Sept./Oct. 1950.

The linear system $dX/dt = (\mu A + \nu B + C)X$, where $X = X(t)$ is a vector, A, B, C are real constant matrixes, and μ, ν are real parameters, has for its characteristic polynomial an expression of the form $\Delta(z) \equiv P(z)\mu + Q(z)\nu + R(z)$. The domain of aperiodic stability is the region in the (μ, ν)-plane in which $\Delta(z) = 0$ has all its roots real and negative. The discriminant curve Y is useful for the study of the domain of aperiodic stability; Y is given in parametric form by the equations $P(\epsilon)\mu + Q(\epsilon)\nu + R(\epsilon) = 0 = P'(\epsilon)\mu + Q'(\epsilon)\nu + R'(\epsilon)$, where ϵ is a real parameter. If a point (μ, ν) crosses Y in an "appropriate direction," a pair of complex roots of $\Delta(z) = 0$ disappears. The main theorem characterizes this direction precisely.

Courtesy of Mathematical Reviews J. G. Wendel, USA

1939. **Doetsch, G., Handbook of the Laplace transformation. Vol. I. Theory of the Laplace transformation [Handbuch der Laplace-Transformation. Bd. I. Theorie der Laplace-Transformation]**, Basel, Verlag Birkhäuser, 1950, 581 pp.

This well-written, well-documented book should appeal to either the mathematician or mathematically trained engineer or the physicist. It is primarily a reference work rather than a text, covering the theory and general properties of the Laplace transform much more extensively than the commonly used American texts, e.g., Churchill's book. This theory is given a generality which should appeal to the mathematician by the use of Lebesgue integration throughout, and the general tone is mathematical (as in Widder's book). However, for the engineer, the proofs are presented in such a way that he can almost always substitute Riemann integrals in the argument, warning being given when this would invalidate the proofs. Some of the particularly worthy special features of the presentation are: A collection in an appendix of certain required theorems with which the reader is not expected to be familiar; mention of presently unsolved problems, helping the reader to distinguish between the unknown and facts merely beyond the scope of the work.

The present work is an extension of the author's earlier book, including progress since 1937 and redirecting emphasis along certain lines. Among other subjects, it extends considerably the earlier treatment of the bilateral transform and the convolution and inversion integrals. Applications are omitted but are to be given in a second volume.

In general, the subject matter is the ordinary unilateral Laplace transform; however, the bilateral transform is discussed and is used to introduce the Mellin transform. The Fourier transform is also developed as a special case of the bilateral Laplace. Some theorems on the unilateral Laplace-Stieltjes transform are given by way of orientation for further study elsewhere. Specifically, the subject matter of part I includes the analytical properties of the transformation and the function-theoretic properties of the transform. Here there is an exhaustive treatment of convergence so often glossed over, and of the well-known theorems on differentiation and integration of the transform, convolution, and the like. Part II concerns the inversion integrals for Fourier and Laplace transforms and the Parseval equation. Part IV discusses the transforms of special classes of functions and includes a chapter on the bilateral Laplace or Mellin transforms. These three parts probably form the meat of the book for the nonmathematician. Parts III and V are devoted, respectively, to a generalization of the Laplace transform and to Abelian and Tauberian theorems. The latter, which concern asymptotic behavior of functions and their transforms, do, however, contain some theorems of considerable interest in physical applications.

Robert E. Roberson, USA

1940. **Neville, E. H., Jacobian elliptic functions, 2d ed.**, New York, Oxford University Press; Oxford, at the Clarendon Press, July 1951, xvi + 345 pp. $7.

This is not the place to discuss various possibilities in the analytic treatment of elliptic functions, beyond that suggested by the title. Let it suffice to remark that the reader already familiar with the elementary parts of complex variable theory will find here a careful, complete, and enjoyable development of elliptic functions, containing all that is required for applications. No actual applications are included; this book belongs to what engineers call pure mathematics, what American pure mathematicians call applied mathematics, since it deals with explicit formulas and calculations concerning particular functions. The reader must not expect to use the work as a handbook; rather, like any sound treatment of this subject, it must be carefully worked through. The length of the book is explained in part by the fact that the ubiquity of the functions is proved without use of Riemann surfaces—to develop whose theory a still longer work would have been required.

The reviewer must take this occasion to deplore current publishing practice. The author of this valuable and unique treatise tells us that in this second edition, lithoprinted from corrected sheets of the first, he was, unfortunately, unable to make more than a small fraction of the improvements he wished.

C. Truesdell, USA

1941. **Gay, P., and Hirsch, P. B., A non-destructive x-ray method for the determination of the thickness of surface layers**, *Brit. J. appl. Phys.* **2**, 8, 218–222, Aug. 1951.

Surface layer may be on single crystal or polycrystalline base. No removal is needed for determinations. Method employs absorption of both incident and reflected beam through the layer. Layer and reflecting planes of specimen determine angle ϕ. Reflecting planes and beam determine Bragg angle θ. Thickness t is the unknown of an exponential equation between these angles, incident and reflected intensities, beam width, and three known coefficients. Author recognizes that uncertainty of absorption coefficient of layer is a disadvantage and that it must be different from matrix coefficient. Spectrometer and Geiger counter are used for measurements. Paper explains process of calibration.

Reviewer has consulted Professor Galloni who considers the method to be original, useful, and proficient.

Jorge Carrizo Rueda, Argentina

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 1934, 1959)

1942. **Stephenson, R. J., Mechanics and properties of matter**, New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1952, x + 371 pp. $6.

This textbook is intended for students of engineering and physics who have had a minimum of a year's course in general physics and the same in calculus. Emphasis is placed on the physical concepts, and mathematics plays the role of a servant. The chapters deal with kinematics, motion of a particle, work and energy of particles, Newton's law of gravitation and some of its consequences, free and forced harmonic oscillations, translational and rotational motion of rigid bodies, elasticity, statics, hydrostatics and hydrodynamics, and wave motion. Drawings are clear and illustrative examples are scattered throughout the text. Exercises are given at the end of each chapter; answers, however, to most of these are not given. This book should give the student further insight into classical mechanics and prepare him for more advanced work in modern physics.

E. J. Scott, USA

1943. Nekrasov, A. I., Course in theoretical mechanics. Vol. I. Statics and kinematics [Kurs teoreticheskoi mekhaniki. t. I. Statika i. kinematika] 4th ed., Moscow-Leningrad, Gosud. Izd. Tekh.-Teor. Lit., 1950, 388 pp.

Under the subject of statics, author discusses concepts of force, moment, vector and scalar products, concurrent and parallel force systems. Then follows treatment of equilibrium of parallel forces, theory of couples, plane systems of forces, friction, rolling and end journal resistance, and arbitrary space force systems. This is done by analytical methods using vector representation which author believes is the best approach.

Next comes graphostatics with discussion of string polygons, moments of forces, and inertia. Applications of string polygon to finding reaction forces and moments in beams are made. A brief discussion of finding forces in truss members by Ritter's (method of sections), Culmann's, and Maxwell-Cremona methods completes the statics part of book.

Kinematics then receives considerable attention. Subjects treated are velocity and acceleration of a point; translation, rotation about a fixed axis, and plane motion of rigid bodies followed by discussion of rotation of rigid bodies about a fixed point. Addition of linear and angular velocities and reduction to simple system are discussed.

The last two chapters are devoted to the general case of motion of rigid bodies, velocity and acceleration of a point in compounded (Coriolis) motion. At this point, moving reference axes receive attention. The entire treatment of kinematics is by vector representation. S. Sergev, USA

1944. Slezkin, N. A., Basic equations of motion of a deformable medium of particles with variable mass (in Russian), *Dokladi Akad. Nauk SSSR* (*N.S.*) **79,** 1, 33–36, July 1951.

Equations of motion, change of mass, and change of momentum of a deformable medium with variable mass are derived. The (vectorial) equation of motion contains the reactive stress tensor in addition to the usual stress tensor, while the equation of change of momentum includes the density tensor of momentum flow of added masses. For a particular form of these equations in the case of a gas, see AMR **5,** Rev. 1078. Some possible applications to the study of turbulent motion of a liquid or a gas are indicated. E. Leimanis, Canada

1945. Shulgin, M. F., Generalization of the Poisson theorem to the case of holonomic nonconservative systems (in Russian), *Dokladi Akad. Nauk SSSR* (*N.S.*) **81,** 1, 23–26, Nov. 1951.

Let the Hamiltonian equations of a nonconservative holonomic dynamical system be

$$\dot{q}_i = \partial H/\partial p_i,\ \dot{p}_i = -\partial H/\partial q_i + Q_i,\ (i = 1, \ldots, n) \qquad [1]$$

where H and Q_i are known functions of the coordinates q_i, the impulses p_i, and time t. Introduce supplementary coordinates u_i and impulses s_i and put

$$K = \Sigma_i(\partial H/\partial p_i)s_i + \Sigma_i(\partial H/\partial q_i - Q_i)u_i$$

Then, with $K(t;\ q_i;\ p_i;\ u_i;\ s_i)$ as the Hamiltonian function, the system [1] assumes the form

$$\dot{q}_i = \partial K/\partial s_i,\ p_i = -\partial K/\partial u_i,\ (i = 1, \ldots, n) \qquad [2]$$

and the u_i and s_i are determined by the equations

$$\dot{u}_i = \partial K/\partial p_i,\ \dot{s}_i = -\partial K/\partial q_i,\ (i = 1, \ldots, n) \qquad [3]$$

The results of the paper can be summarized as follows: Let $f(q_i;\ p_i;\ t) =$ const and $\varphi(q_i;\ p_i;\ u_i;\ s_i;\ t) =$ const denote two initegrals of the system [1] and of the extended system [2] and [3], respectively. Then, if the substitution $u_i = \partial f/\partial p_i$, $s_i = -\partial f/\partial q_i$ is made, the equation $(f, \varphi) =$ const, where (f, φ) denotes the Poisson's bracket-expression of the functions f and φ, constitutes, in general, a new integral of the original system [1]. If, in particular, $f(q_i;\ p_i;\ t) =$ const is an integral of a nonconservative scleronomic system [1], then $\partial f/\partial t =$ const is also an integral of this system, and consequently $\partial^2 f/\partial t^2 =$ const, and so on, are integrals.

The theory is illustrated by an example of a free motion of a material point with unit mass in a resisting medium (the resistance being proportional to the velocity), and attracted by a force proportional to the distance from the origin. E. Leimanis, Canada

1946. Varshavskiĭ, L. A., On the conditions of stability of a linear system (in Russian), *Zh. tekh. Fiz.* **21,** 8, 907–919, Aug. 1951.

Author notes that, although the Hurwitzian criteria of stability permit ascertaining whether a given system is stable or not, the inverse problem, i.e., the determination of the coefficients in advance so as to secure a stable system, presents some difficulties. His purpose is precisely to attempt to obviate this difficulty. It is noted that, in the general case of a linear system with many degrees of freedom, the variables are determined in the form of rational fractions involving certain polynomials. Author applies the well-known property of polynomials according to which the simple (real or imaginary) roots of polynomials formed by the even and by the odd terms of the original polynomial, alternate in the sequence of roots. The establishment of these conditions occupies the major part of the paper, and it is shown that the problem reduces to the determination of certain stable regions in the multi-dimensional space of the parameters. For the general case of n degrees of freedom, the calculation is complicated and cannot be abstracted. It is sufficient to mention that use is made of certain properties of determinants in which some columns are omitted. N. Minorsky, France

1947. Goodman, T. P., Toggle linkage applications in different mechanisms, *Prod. Engng.* **22,** 11, 172–173, Nov. 1951.

1948. Chebíshev, P. L., Theory of the mechanisms known by the name parallelogram. With commentaries by V. L. Goncharov, I. I. Artobolevskiĭ, and N. I. Levitskiĭ, and the addition of an essay by I. I. Artobolevskiĭ and N. I. Levitskiĭ on the development of approximate methods in the synthesis of mechanisms according to Chebíshev. [Teoriya mekhanizmov izvestníkh pod nazvaniem parallelogramov. S kommentariyami V. L. Goncharova, I. I. Artobolevskogo, i N. I. Levitskogo, i prilozheniem stat'i I. I. Artobolevskogo, i N. I. Levitskogo, razvitie priblizhenníkh metodov sinteza mekhanizmov po Chebíshevu], Moscow-Leningrad, Izdat. Akad. Nauk SSSR, 1949, 80 pp.

As a contribution to the current Chebíshev revival, his 1853 paper ["Théorie des mécanismes connus sous le nom de parallélogrammes," *Mém. Acad. impériale Sci. St.-Pétersbourg* **7,** 537–568, 1854] has been republished (in an 1899 Russian translation). ("Parallelogram" stands here for "linkage.") No specific applications to linkages are presented in the paper, which is purely mathematical, except for the introduction which shows the technological origin of the problem of best approximation. The principle of full oscillation (credited to Poncelet) is merely enounced and applied to some approximation problems by means of power polynomials (34 pp.).

The commentary by Goncharov (16 pp.) is a popular lecture on the Watt linkage (with puzzling wealth of detail), followed by a mention of the credit due to Poncelet for the principle of full

oscillation, a brief and restricted proof of it, and a few examples of best approximation. The commentary by I. I. Artobolevskiĭ and Levitskiĭ (16 pp.) is simply a summary of Chebíshev's papers (with some direct quotations). The appendix by the same two authors is another survey of Soviet work based on Chebíshev's approximation [see *Akad. Nauk SSSR Trudĭ Sem. teor. Mash. Mekh.* 4, 16, 5–46, 1948]. A. W. Wundheiler, USA

1949. **Artobolevskiĭ, I. I., Foundations of the kinematics of automatic machines** (in Russian), *Trudĭ Sem. teor. Mash. Mekh.* 1, 81–126, 1947.

Author introduces some terminology and some logic for the discussion of the general design of automatic machinery. His concern is the timing and duration of the component processes involved. For reciprocating parts, the dependence of the duration of the cycle on the law of acceleration is discussed. Some routines (graphical and others) for the setting up of a timing plan and of activating elements are described, also in the case of several driving shafts. A. W. Wundheiler, USA

1950. **Artobolevskiĭ, I. I., A new method for the determination of flywheel masses** (in Russian), *Trudĭ Sem. teor. Mash. Mekh.* 1, 49–56, 1947.

Author first derives the formula

$$\omega_r^2 dI_r/dt = 2\Sigma[mva + I\omega\epsilon]$$

where ω_r is the constant angular speed of the driving member, $I\omega_r^2/2$ the kinetic energy of the machine, v and a the speed and tangential acceleration of the mass center of a link, ω and ϵ its angular velocity and acceleration, and m, I its mass and central moment of inertia. Then he suggests a certain correction for the case of variable ω_r, which is based on the extreme values of ω_r and I. He argues that his method is superior to the graphical ones of Wittenbauer, Mertsalov, and Radinger.

A. W. Wundheiler, USA

1951. **Levitskiĭ, N. I., Design of plane mechanisms with lower pairs [Proektirovanie ploskikh mekhanizmov s nizshimi parami]**, Moscow-Leningrad, Inst. Mashinostroeniya, 1950, 182 pp.

This is a monograph on the method of least squares in approximate linkage design. The method also appears in combination with the method of linear corrections. The author's own results constitute the bulk of the book. The problem is to generate a function or to trace a curve; the mechanisms are the four-hinge linkage and the simple slider-crank linkage. All solutions are followed by numerical illustrations. The presentation is very explicit.

An introductory chapter 1 discusses the method and the mathematical prerequisites. In chapter 2 the four-hinge linkage $ABCD$ is described by means of six parameters (three side ratios, the direction of the stand AD, and the limits of the working-range interval); the relation between the lever angle ψ and the crank angle α is the generated function. Assuming C to be a crosshead C_x and ψ the required function of α, $\Delta_q = BC_x^2 - BC^2$ is the accepted measure of the deviation. It is given the form

$$F(\alpha) - p_0\varphi_0(\alpha) - p_1\varphi_1(\alpha) - p_2\varphi_2(\alpha)$$

and p_0, p_1, p_2 are determined to make $\int \Delta_q^2 d\alpha$ a minimum over the working range. It can also be represented in the form $F(\alpha) - p_0\varphi_0(\alpha) - p_1\varphi_1(\alpha) - p_2\varphi_2(\alpha) - p_0p_1\varphi_4(\alpha)$, and then p_0, p_1, p_2, p_3 can be determined. The functions F and φ are not uniquely determined, and have been chosen so as to make them dependent on α and the given parameters of the mechanism. Worked-out examples show how linear corrections can refine the approximation still further. The functions generated here are $\psi = \alpha$ and $\psi = \log \alpha$. Also, the "best," i.e., uniform (= Chebíshev: equal extreme deviation) approximation is obtained (as a further refinement) by successive linear corrections. All this work is numerical.

In chapter 3, for curve tracing by means of a connecting-rod point M, C is again replaced by a slider crank and $\Delta_q = DC_x^2 - l^2$ is accepted as the measure of the deviation when the tentative point M is moved upon the curve to be traced. Δ_q is treated as in the preceding chapter. Numerical examples deal with a ballistic curve and a certain closed curve, both given by means of a finite number of points. The problems of three, four, and five parameters are treated, the latter again by applying successive linear corrections.

In chapter 4, the mechanism is a slider-crank linkage ABC, and the abscissa X of the crosshead C is a function of the angle α of the crank AB, used to generate the given function. Again if BC is assumed to be variable, $\Delta_q = BC^2 - l^2$ is accepted as a measure of the deviation if X is the given function of α. The same method of least square deviation is used for the determination of two, three, and four parameters. The numerical work deals with the function $X = \tan \alpha$.

Chapter 5 takes up curve tracing by means of a point of the connecting rod of the slider-crank mechanism. Here $\Delta_q = AB^2 - l^2$ (AB – crank) is assumed as the measure of the deviation. The previously considered ballistic curve is used for numerical work. A bibliography of 100 titles is appended. It contains only seven non-Russian references, of which only one [Svoboda, AMR 2, Rev. 1228] is in English. A. W. Wundheiler, USA

1952. **Levitskiĭ, N. I., Design of a four-bar linkage with four and five variable parameters, tracing a given trajectory** (in Russian), *Trudĭ Sem. teor. Mash. Mekh.* 7, 27, 5–38, 1949.

1953. **Levitskiĭ, N. I., Design of a four-bar linkage for a given motion of the following crank** (in Russian), *Trudĭ Sem. teor. Mash. Mekh.* 7, 28, 26–73, 1949.

1954. **Levitskiĭ, N. I., and Levitskaya, O. N., Design of slider-crank linkages** (in Russian), *Trudĭ Sem. teor. Mash. Mekh.* 8, 31, 5–46, 1950.

These papers have been incorporated in the first author's book (see preceding Rev. 1951), as pages 121–140, 47–92, and 160–175, respectively. The least-squares method, together with an idea of deviation measurement, are applied, as described in the review of the book. A. W. Wundheiler, USA

1955. **Geronimus, Ya. L., On the application of Chebíshev's methods to the problem of the balancing of mechanisms. [O primenenii metodov Chebísheva k zadache uranoveshivaniya mekhanizmov]**, Moscow-Leningrad, Gosud. Izdat. Tekh. Teor.-Lit., 1948, 148 pp.

The first chapter (52 pp.) of the book presents a survey of some approximation methods (interpolation, expansions, least-squares, Chebíshev's uniform approximations combined with successive ones) with special attention to approximate expressions of the form $\Sigma c_k\zeta_k$, $\zeta_k = \exp(ik\alpha)$ with complex c_k. A graphical method is given for the last mentioned method. The remainder is an expansion and an elaboration of the author's previously published ideas [AMR 5, Revs. 981, 982]. Mechanisms containing members rotating about axes perpendicular to the crankshaft are considered.

The mathematical feature of the method is the use of complex numbers to represent the main inertial force p' and the transverse inertial moment m'_{yx} of the crankshaft: $p' = R\omega^2\Sigma c_k\zeta_k$, $m'_{xy} = R\omega^2\Sigma d_k\zeta_k$. The axial inertial moment is real: $M_z' = R\omega^2\Sigma(g_k \cos k\alpha_k + h_k \sin k\alpha)$. The origin of the moments is the center of symmetry of the shaft, α is the crank angle, ω the angular speed.

The counterweights of masses m are assumed to rotate about axes parallel to the crankshaft. Their inertial main forces can be represented by $\mu R\omega^2\zeta$, and inertial moments by $i\mu z R\omega^2\zeta$, where $\mu = m \exp(i\phi)$; R, ϕ, z are the cylindrical coordinates of the counterweights. The author distinguishes between "outer" balancing when only the total main force and the total inertial torque are separately minimized (in the least-squares or the Chebíshev sense) and "full" balancing when each of the bearing reactions is minimized.

Chapter 2 (27 pp.) is devoted to the problem of outer balancing by means of two counterweights. Both least-squares and Chebíshev minimizing are considered. If the counterweights are solid with the shaft, only transverse balancing can be achieved. If they are made to rotate about axes parallel to the shaft, axial minimal balancing can be achieved as well. The results are extended to the case in which each throw of the crankshaft controls the same mechanism (as in engines of the types V, W, or X). It is shown that the solution of this problem can be easily derived from that of the balancing of the crankshaft alone, by multiplying the masses of the counterweights by the same factor, and turning them by the same angle. A specific application is made to the eccentric of a locomotive.

Chapter 3 (20 pp.) deals with full balancing. There arises here the additional problem of the distribution of the reactions among the bearings, and influence coefficients are determined. Chapter 4 (12 pp.) concentrates on the one-cylinder reciprocating engine. Chapter 5 (16 pp.) considers bearing reactions only (see the second paper cited above) assuming negligible resultant inertia forces. Minima are required for each reaction and for the total mass of the counterweights. Here least-squares minima are used, and gas reactions accounted for.

The novelty of the author's approach consists in going beyond the conventional method of balancing the first or second harmonics (in the crank angle) of the inertial forces.

A. W. Wundheiler, USA

1956. Semenov, M. V., Design of partly balanced plane mechanisms (in Russian), *Trudi Sem. teor. Mash. Mekh.* **8**, 29, 74–90, 1949.

Author considers the balancing of the first harmonic of the resultant inertia of a plane mechanism, by modifying only the mass distribution of some working members. For a four-bar or slider-crank linkage in one plane, he does it by moving the mass centers of the cranks, or of one crank and the connecting rod. Other examples are considered before mechanisms having several parallel planes are considered. The equations are simple and fully treated.

A. W. Wundheiler, USA

1957. Semenov, M. V., Balancing of spatial mechanisms (in Russian), *Trudi Sem. teor. Mash. Mekh.* **8**, 32, 31–42, 1950.

It is shown, for rotating machinery in space, that if the 12 coefficients of the kth harmonic of the inertia force are given, three balancing counterweights can be easily determined (they must be mounted on conical gears rotating at the speed $k\omega$). For the case of the first harmonic, author extends method of the preceding paper to the spatial case. A. W. Wundheiler, USA

1958. Catlow, Marjorie G., and Vincent, J. J., The problem of uniform acceleration of the shuttle in power looms, *J. Text. Inst. Trans.* **42**, 11, T413–T488, Nov. 1951.

Authors present a comprehensive theoretical investigation of shuttle movements for rationalizing designs of cam-operated picking mechanisms so as to reduce maximum accelerating force involved in picking. Various new movements are proposed and mathematically analyzed, including those shown in British patent 642,856. Results are exhibited graphically throughout the paper.

In comparing nominal and actual performance of the movements, the effects of "alacrity" of the mechanical systems and other loom parameters are taken into account. Sensitivity of the proposed movements to loom speed and to chance variations in rigidity of the mechanism is considered. Alacrity is defined as the ratio of the mass of the shuttle and picker to the rigidity of the system.

Methods employed are, first, to postulate the form of nominal movement and then deduce actual conditions of acceleration; second, to impose on the actual acceleration a function of time, thus determining the nominal movement.

Criteria for realizing the advantages of the new movements are pointed out. Authors conclude that the reduction in maximum accelerating force at a given velocity is governed by the distance available in the shuttle box and the alacrity of the system.

Lloyd R. Koenig, USA

Gyroscopics, Governors, Servos

(See also Revs. 1930, 1938)

1959. Bromberg, P. V., On the problem of stability of a class of nonlinear systems (in Russian), *Prikl. Mat. Mekh.* 14, 5, 561–562, Sept./Oct. 1950.

Author obtains a slight extension of the stability criteria given by Lur'e [AMR **3**, Rev. 829] for the system

$$\dot{x}_s = \lambda_s x_s + f(\sigma),\ s = 1, \ldots, n,\ \dot{\sigma} = \Sigma_1^n \beta_\alpha x_\alpha - rf(\sigma)$$

In particular, the author is able to treat the case when $r + \Sigma_1^n \beta_\alpha \lambda_\alpha = 0$, to which the Lur'e method is inapplicable.

Courtesy of Mathematical Reviews J. G. Wendel, USA

1960. Ahrendt, W. R., and Taplin, J. F., Automatic feedback control, New York, Toronto, London, McGraw-Hill, 1951, xiv + 412 pp. $7.50.

This book forms a bridge between some of the modern books on the theory of servomechanism and the older publications on automatic control engineering. The book is on an intermediate level of difficulty and successfully integrates the practical problems of industrial control with the recently developed mathematical techniques of feedback-systems analysis.

Chapter 1 introduces the reader to the subject matter. Chapter 2, "Dynamical analysis of physical systems," reviews the physical and mathematical background required for transient and steady-state analysis. A review of the "Theory of Laplace transforms" is included. Chapters 3–6 present the general theory of linear feedback-control systems. In particular, chapter 3 presents transient analysis of some simplified systems. Chapter 4 presents an introduction to steady-state analysis, including Nyquist criterion. Chapter 5 presents the steady-state analysis of various feedback-control loops, including the effects of time lags, proportional plus integral control, proportional plus lead control. The use of log-decibel charts and various phase-gain relationships are also presented. Chapter 6 deals with multiple-loop control systems.

Chapter 7 discusses such design considerations as the characterization of controlled systems and disturbances on the basis of empirical data. Chapter 8 deals with nonlinearities such as (1) time variable elements, (2) variable gain, (3) friction and backlash, (4) saturation, (5) discontinuous controls.

The remainder of the book applies the techniques previously developed to the design and analysis of (1) positioning devices,

(2) pneumatically operated controllers, (3) temperature regulations, (4) speed governing, (5) pressure, flow and liquid level control. These three chapters comprise about one third of the book and present a systematic and practical discussion of these feedback-control problems of wide industrial application.

At the end of the book there is a large number of problems. These should be welcome by all those who intend to acquire a working knowledge of feedback controllers.

Andrew Vazsonyi, USA

Vibrations, Balancing

(See also Revs. 1931, 1942, 2074, 2236)

1961. **Pignedoli, A., On the effective determination of the vibration frequencies of a homogeneous plate with an epicycloidal built-in contour** (in Italian), *Atti Semin. Mat. Fis. Univ. Modena* 4, 30–44, 1949–50.

Frequencies are obtained from determinant of infinite order, each element being a sum of integrals of products of Bessel and trigonometric functions. No suggestions are given as to practical numerical procedures.

William A. Mersman, USA

1962. **Johnson, D. C., A new treatment of the tuned absorber with damping,** *Aero. Quart.* **3,** part 3, 182–192, Nov. 1951.

Author considers the problem of the optimum design of a torsional vibration damper, which includes a linear viscous damping, for use with multiple-cylinder engines. This problem has been treated by Timoshenko ["Vibration problems in engineering," 2nd ed., 1937, p. **245**] in an oversimplified form. The problem has been treated in more generality by Zdanowich and Moyal [*Instn. mech. Engrs. Proc.* **153,** 1945].

Author's treatment is similar to the second of these, but puts the design on a graphical basis. Starting with the concept of an "admittance" in the sense of Duncan, i.e., the ratio of deflection to force, author builds up relations for admittances connected in parallel. He then shows how these relations can be nondimensionalized and plotted as three-dimensional graphs. From these graphs, the optimum design can be found by noting points of minimum separation between specified curves.

It is interesting to note that formulas appear in the form of reciprocals of admittances, which suggests that the treatment would be simpler in terms of mechanical impedances, as usually defined.

Horace M. Trent, USA

1963. **Fifer, S., Studies in nonlinear vibration theory,** *J. appl. Phys.* **22,** 12, 1421–1428, Dec. 1951.

Author treats stability conditions for second-order first-degree nonlinear equations by means of a variational procedure. Stability of the Duffing equation is considered both with and without a damping term. Stability of the van der Pol equation is also considered with the damping term represented as an odd polynomial. Several response curves are presented indicating regions of stability and instability for the van Pol equation. A so-called vertical tangent theorem is proved which states that "the locus of the points of contact of the vertical tangents to the response curves coincides in the first approximation with the locus of periodic solutions of the variational equation." Two diagrams outline stability regions in three dimensions for the van der Pol equation with first-, third-, and fifth-degree damping terms.

Will J. Worley, USA

1964. **Richards, J. E., An analysis of ship vibration using basic functions,** *N. E. Cst. Instn. Engrs. Shipb. Trans.* **68,** part 2, 51–92, Dec. 1951 = *Shipbuilder* **59,** 524, 293–299, Apr. 1952.

Natural frequencies of the vertical vibrations of ships are calculated, and results are compared with previously reported data [AMR **4,** Rev. 2364]. Natural frequencies are obtained by direct solution of the differential equation for the free-free vibration of a beam of variable density and moment of inertia; it is assumed that the deflection curve is representable by superposing a few of the normal modes of a uniform beam; allowance is approximately made for the effects of shear deflection and rotary inertia.

Author concludes that the excess of the calculated frequencies over the observed [op. cit.] frequencies of the three- and four-node modes is not due to lack of refinement of the calculations, but rather to the ship's failure to behave as a beam. This conclusion was substantiated by further tests on the same vessel, in which there were revealed superposed vibrations of the double bottom having an amplitude greater than that of the deck by a factor of from 3 to 10.

Sanford P. Thompson, USA

1965. **Zheleztsov, N. A., The method of point transformation and the problem of the forced vibrations of an oscillator with "combined friction,"** *Amer. math. Soc. Transl.* no. 57, 49 pp., 1951.

See AMR **3,** Rev. 217.

Wave Motion, Impact

(See also Revs. 1981, 2013, 2124, 2228)

1966. **Biot, M. A., The interaction of Rayleigh and Stoneley waves in the ocean bottom,** *Bull. seism. Soc. Amer.* **42,** 1, 81–93, Jan. 1952.

A theory is developed for the propagation of two-dimensional unattenuated waves in a system consisting of a liquid layer overlying an infinitely thick solid. Special attention is given to the interaction between the Stoneley type of wave and the Rayleigh wave. It is shown that the type of wave discussed corresponds to a dispersion branch for which the velocity varies continuously from a value lower than the velocity of sound in the liquid to that of the Rayleigh waves. The possible importance of this fact is pointed out in connection with the interpretation of the T phase of shallow-focus submarine earthquakes. The physical nature of these waves is illustrated by showing that they exist at the interface of a massless solid and an incompressible fluid.

From author's summary by J. K. Lunde, Norway

1967. **Gröbner, W., Surface waves of liquids** (in German), *Ann. Scu. norm. sup. Pisa* (III) **5,** 3/4, 175–191, July/Dec. 1951.

Author considers an application of variational calculus to problem of water waves. In contrast with usual method of equation of motion, potential and kinetic energy are first calculated. After applying the variational principle to the total energy, a differential equation is obtained, the elevation of water surface being obtained by solving it. Though author obtains a general form for any shape of bottom, actual solutions are given for horizontal and inclined plane bottoms. Author asserts that method gives a higher approximation than usual method. Though author's method seems ingenious, reviewer thinks that some neglecting of higher-order terms cannot be avoided in calculating energy in actual problems.

Tsuneji Rikitake, Japan

1968. **Jeffreys, H., On the highest gravity waves on deep water,** *Quart. J. Mech. Appl. Math.* **4,** part 4, 385–387, Dec. 1951.

Author discusses method of Michell and Havelock for the determination of the highest possible progressive wave on deep water. It is known that, from theory, Michell and Havelock give

two methods for estimating the departure of the slope from uniformity near a crest and on one side of it, and the results do not agree. This paper shows that difference is probably due to the retention of a small number of terms in the series.

Reviewer remarks that an indirect proof of this point of view is given by Davies [AMR **5**, Rev. 355] who obtained Michell's results by a new method. Giulio Supino, Italy

1969. Lee, E. H., and Wolf, H., Plastic-wave propagation effects in high-speed testing, *J. appl. Mech.* **18**, 4, 379–386, Dec. 1951.

A material test carried out at high speed may be markedly influenced by plastic-wave propagation effects. In such a case, a variation of strain occurs along the test specimen, and the stress-strain relation cannot be determined from measurements made on the specimen as a whole. If average values are taken, it is shown that a spurious strain-rate influence will be deduced when propagation effects first begin to appear as the testing speed is increased. The effect is due not to a true material strain-rate dependence, but to the appearance of strain variations along the specimen, and its magnitude depends upon the dynamics of the whole test arrangement. The theoretical plastic-wave analysis of a particular test arrangement is also given. The range of speed is determined which permits satisfactory interpretation without the need for detailed analysis of plastic-wave propagation. Application to other test arrangements is discussed, and it is pointed out that a theoretical analysis can, in general, be made to predict the permissible speed range in planning a high-speed testing program.

From authors' summary by C. O. Dohrenwend, USA

Elasticity Theory

(See also Revs. 1925, 1983, 1986, 1995)

1970. Prokopov, V. K., On a two-dimensional problem of theory of elasticity for a rectangular region (in Russian), *Prikl. Mat. Mekh.* **16**, 1, 45–56, Jan./Feb. 1952.

Two-dimensional problem of bending of a beam of narrow cross section, with two built-in edges, loaded by a force P, has a known solution given in form of a sum of an algebraic polynomial and a Fourier series of real arguments. But in this way, on built-in edges, one can satisfy only the relaxed condition in which a horizontal or vertical element in the axis of beam has zero displacements. In this paper, problem is solved under a more severe condition in which displacements in direction of the axis of beam of all points of built-in edges are zero. For this purpose, using a method proposed by Papkovitch [*Dokladi Akad. Nauk SSSR*, **27**, 4, 1940], a biharmonical function in form of a complex series, satisfying homogeneous conditions on upper and lower edges, is added to the known solution. Coefficients of this series are determined so as to satisfy conditions on built-in edges; this requires developing expressions given by hyperbolic functions in terms of some nonorthogonal functions of complex argument, which is achieved by applying theorem of residua and by use of some summation formulas. Then, the complex series representing the additional biharmonical function is converted into a real one, also by use of theorem of residua, which makes the solution suitable for numerical calculation. Solution for arbitrary loading on the beam can be obtained from the above by integration. Finally, it is shown that, in case of small height of beam, this solution reduces to the elementary solution given in "Strength of materials." Dragoš Radenković, Yugoslavia

1971. Schmidt, K., Treatment of plane elasticity problems by means of hypercomplex singularities (in German), *Ing.-Arch.* **19**, 6, 324–341, 1951.

The identity between the differential equations satisfied by the stress components of two-dimensional elastic systems and the components a, c, d of a hypercomplex function $f = a + jb + j^2c + j^3d$ with $1 + 2j^2 + j^4 = 0$, established by L. Sobrero [*Memor. reale Accad. d'Italia* **6**, p. 1, 1934–35], is applied to the solution of plane stress problems with concentrated forces. The investigation is facilitated by the use of a generalized method of images. For a concentrated force inside an infinite or semi-infinite plate, the well-known results are obtained. Solution in the form of infinite series is given for a concentrated force parallel to the sides of an infinite strip, and author claims good agreement of numerical results with the solutions of E. Melan and K. Girkmann for the special case of a load halfway between the edges of the strip. The problems of two equal and opposite forces applied either along the diameter of a circular disk or of an infinite plate with circular hole are also solved. For the latter case with the diameter of the hole much smaller than the distance of the points of application of the forces, solved otherwise by R. Sonntag, the solution is given in closed form. Sonntag's results for forces applied at the ends of a diameter of the hole are also reached. Author mentions, without comparative evaluation, the well-known methods of analytic functions of complex conjugate variables developed by N. Muskhelishvili and of complex potential by A. C. Stevenson. Reviewer remarks that author's method has certain advantages of compactness but requires the use of the rather complicated algebra of hypercomplex numbers and lacks in elegance when compared to the other two methods mentioned.

George A. Zizicas, USA

1972. Szebehely, V. G., and Pletta, D. H., The analogy between elastic solids and viscous liquids, *Virginia polyt. Inst. Bull.* **45**, 1, 24 pp., Nov. 1951.

Authors discuss in tensor form the behavior of elastic solids and viscous (Newtonian) fluids in the case where linear relations exist between (1) displacement and strain tensors, (2) strain and stress tensors (Hooke's law), and (3) velocity derivatives and stress tensors. In that case the two are completely analogous, although the analogy is purely formal. Paper is well written but does not contain any new results [see, e.g., Love, "Elasticity"; Lamb, "Hydrodynamics"; Alfrey, "Mechanical behavior of high polymers"]. D. ter Haar, Scotland

Experimental Stress Analysis

(See also Rev. 2045)

1973. Koch, J. J., Boiten, R. G., Biermasz, A. L., Roszbach, G. P., and van Santen, G. W., Strain gauges; theory and application, Eindhoven, Holland, Philips Techn. Library, 1952, 95 pp. \$2.75.

Book is limited essentially to a discussion of bonded-wire electrical resistance strain gages of the type (constantan wire bonded with cellulose cement) manufactured by Phillips-Eindhoven, and of instrumentation manufactured for use with such gages. Chapter headings are: (1) Construction and properties of strain gages; (2) Measuring instruments; (3) Technique of cementing and connecting strain gages; (4) Evaluation and interpretation of the values measured with the aid of strain gages in measuring instruments.

Most of the discussion is clear and readable. Suggestions such as those in chapters 3 and 4 should be helpful to a beginner in using wire strain gages. To this reviewer, chapter 5 seems written more abstractly than other chapters, and part of the material (in regard to theories of failure) seems irrelevant to a book on strain gages and strain measurement.

Horace J. Grover, USA

1974. **Boni, M., On the use of brittle lacquers in the determination of stresses** (in Italian), *Atti Ist. Costr. Univ. Pisa, Pubbl.* 22, 8 pp., 1951.

Author describes some brittle coating tests he conducted on a square plate under a concentrated load at the center, and supported at the four corners. Separately, he develops a finite difference solution of the same case. The paper is sketchy and there is no correlation of results between theoretical and experimental solutions. The coating is used to determine isostatics only. The photographs are very clear. A. J. Durelli, USA

1975. **Durelli, A. J., and Ōkubo, S., Heat treated brittle coating increases sensitivity,** *Prod. Engng.* 22, 12, 144–147, Dec. 1951.

Stresscoat is normally intended for investigation of areas of high strain. When used for exploring understressed zones, sensitivity is increased usually at risk of temperature crazing. Authors report results of a heat-curing procedure for increasing sensitivity. Method involves drying 5 minutes at room temperature, curing in oven at elevated (75 F to 135 F in tests) temperature 17 hours, oven cooling 3 hours, air cooling 2 hours. Curves give increase in sensitivity vs. curing temperature. For temperatures above 85 F, sensitivity varies considerably with coating thickness, reaching maximum at about 0.007-in. thickness. Quantitative accuracy thus requires uniform coat. It is interesting to note that for room temperature curing, sensitivity is virtually independent of thickness. D. K. Wright, Jr., USA

1976. **Sciammarella, C. A., and Palacio, M. A., Photoelastic test of a beam of great height** (in Spanish), *Cienc. y Técn.* 117, 589, 1–31, July 1951.

Photoelastic tests by Favre's optical method of simply supported beams with a ratio of depth : span = 1.5 are described, and the results compared with the ratios obtained by theoretical means. A detailed description of the loading device used to achieve a uniformly distributed load and of the support conditions is included. Finally, a comparison of the results of the present test with previous ones for the ratio of depth : span = 1, and some general conclusions regarding the influence of the depth on the stress distribution are drawn. Ernesto Saleme, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 1929, 1976, 1998, 2005)

1977. **de Beer, E. E., and Krasmanovitch, D., Calculation of beams resting on soil. Case of uniformly distributed loads, of equidistant loads, and eccentric loads** (in French), *Ann. Trav. publics Belg.* 104, 6, 981–1014, Dec. 1951.

Memoir is an extension of work published June to December 1948 [see AMR 2, Rev. 1199] by Dr. de Beer alone, in which modulus of subgrade reaction was shown to give values of contact-face equation incompatible either with known soil deformation or with known footing deformation. The suggested solution was to assume a given curve for the soil-footing contact face and to calculate the stress pattern at the surface of the soil and at the base of the footing, and by successive approximations obtain a fair correspondence. The method is now applied to various footing loads, and the results are compared with those obtained by the coefficient-of-subgrade reaction method.

Robert Quintal, Canada

1978. **de Beer, E. E., and Krasmanovitch, D., Calculation of beams resting on soil** (in French), *Ann. Trav. publics Belg.* 105, 1, 9–50, Feb. 1952.

Further explanation and application of theory developed in preceding papers [see AMR 2, Revs. 1199, 1545, 1546, and preceding review]. Authors now establish the stress pattern under a beam loaded eccentrically and resting on soil by finding, by successive approximations, the elastica of the beam which corresponds to the soil deformation.

Robert Quintal, Canada

1979. **Hudler, S., Load distribution in a compound cable** (in German), *Elektr. Masch.* 69, 4, 85–87, Feb. 1952.

An exact method is given for computing the stresses in the individual strands of a long-span, different material cable under the influence of elastic and thermal elongation. It assumes equal total elongation for the different material strands and does not consider creep. A numerical example of an aluminum-steel cable is given. Dimitri Kececioglu, USA

1980. **Anonymous, Design charts for large compression springs,** *Prod. Engng.* 23, 2, 203–207, Feb. 1952.

Charts are for calculating helical springs made of round bars ranging in diameter from $^1/_2$ to $2^1/_4$ in. Ed.

1981. **Yoshimura, Y., and Murata, Y., On the elastic waves propagated along coil springs** (in Japanese), *Rep. Inst. Sci. Technol. Tokyo* 6, 1, 27–35, 1952.

In treating the problem of surging and shock-absorbing action of a coil spring, the velocities of waves propagated along the spring are usually assumed to have constant values $(K/M)^{1/2}$ independent of their wave lengths, where K denotes spring constant for extension or torsion, M the mass per unit length of the spring axis. This assumption, which is equivalent to replacing the spring by a straight rod with the same elasticity and density as the spring, is considered to be approximately correct for very long waves, but the behavior of the spring for such short waves or impulses as the above examples is not known. Paper clarifies these points by analyzing the dispersive characters of the coil spring. According to the results, there are three waves which coexist and couple with each other; their velocities along the wire are v_1, v_2, and v_3, respectively. v_1- and v_2- waves correspond to the longitudinal waves along the axis of the spring, and v_3-wave to the rotational wave. v_1 and v_3 have constant values $(K/M)^{1/2}$ for long wave length, and very small minima at the wave length equal to the coil circumference. For very short wave length, the curvature of the wire may be reasonably neglected.

From authors' summary

1982. **Bozza, G., Hazards of natural gas pipe lines** (in Italian), *Termotecnica* 5, 12, 529–543, Dec. 1951.

Paper concerns hazards associated with transport of natural gas in long pipe lines. Author summarizes some associated flow formulas and discusses measure of corrosion hazard. Based on energy that can be liberated per unit length of tube, hazards due to a slow leak or a sudden rupture are evaluated.

M. J. Goglia, USA

1983. **Föppl-Sonntag, Diagrams and tables for the theory of strength of materials [Tafeln und Tabellen zur Festigkeitslehre],** München, Verlag R. Oldenbourg, 1951, 138 pp. + 305 figs.

Reviewer is particularly impressed with the important and highly valuable part of this book, namely the text, which serves to illuminate and, further, to explain the procedure to follow in using this rich collection of tables, diagrams, nomograms, and sketches. The engineer and scientist will find easily accessible, authentic information embracing the assumptions made in deriving formulas and in preparing diagrams, and pertinent points to consider in

analyzing each case. For explicit illustration of the methods developed, authors have included many solved examples. Each chapter contains an exhaustive bibliography.

This book presents the fundamental theoretical knowledge and wide experience of these eminent authorities in a lucid, easily grasped style. There are four main parts: I: "Undisturbed state of stresses." Bending of straight and curved beams and rings; torsion; two-dimensional stress distribution in disks, wedges, cylinders, infinite plates, eccentric rings with open gaps; three-dimensional stress distribution in a semi-infinite body. Hertz's hardness, pressure between bodies in contact, pressure distribution in bodies on elastic foundation; plate subdivided into (a) those whose deflection is smaller than the thickness, with an extensive treatment by influence areas according to a work by Professor Puchner, (b) plates whose deflection is large, with flexural and axial stresses, (c) membranes, (d) thick plates; thin and thick shells. Part II: "Notch stresses." Two-dimensional notch stresses in flat bars and disks; in plates; three-dimensional circumferential notch stresses; prismatic notch effect; nomograms of stress-concentration factors with several examples solved and application of notches in stress-relieving. Part III: "Stability problems." Buckling of bars and shafts, plates, disks, and shells. Part IV: "Miscellaneous." Information of a general character on the scope and limitation of the classical theory of elasticity; the influence of the size and form of the cross section and the effect of the grip on the endurance strength; thermal stresses in a thin-walled cylinder; flexural impact against a beam and a plate.

This is a book which will further justify the already well-established reputation of its authors for accuracy, authoritativeness, and practicability. Wilhelm Ornstein, USA

1984. Barthélémy, J., Study of the deformation and stresses in curved pipes subjected to external forces and internal pressures (in French), *Bull. Assn. tech. marit. aéro.* no. 50, 589–609, 1951.

Author extends previous theoretical investigation on the bending of tubes [AMR **3**, Revs. 1060, 1061] to include case of plane curved tube bent out of its original plane. One of the most significant results of the analysis is that the coefficient of transverse flexibility is not equal to unity (relative to that for straight tube) but is the same as that for the plane curved tube bent in its plane. The coefficient in the latter case is quite different from unity. A table of comparison of the flexibility coefficients for both cases is provided. The extremely long and tedious equations which serve the author as bases for tables of coefficients are given in appendixes. W. H. Hoppmann, II, USA

1985. Panetti, M., The static and constructional problem of a set of two perpendicular tubes, subjected to internal overpressure and depression, in a wind tunnel with closed circuit (in Italian), *Monogr. Lab. Aero. Politecn. Torino = Pontif. Acad. Scient.* **14**, 9, 87–106, 1951.

The wind tunnel under consideration has a closed rectangular circuit made of four circular concrete tubes. The construction of such a tunnel gives rise to two problems, which are studied successively: (1) The stresses in the connections between two perpendicular tubes are investigated by cutting by two parallel planes an elliptical connection ring; this ring is statically indeterminate and its action is calculated by the principle of least work. (2) The problem of prestressing the concrete tubes in order to prevent any crack under either pressure or depression is solved by comparing different dispositions of the reinforcing wires. The classical reinforcement made of longitudinal and circumferential wires is compared to another system of wires whose meshes form lozenges. Ch. Massonnet, Belgium

1986. Nardini, R., On the elastic line of a compressed-bent beam in the presence of hereditary phenomena (in Italian), *R. C. Semin. mat. Univ. Padova* **20**, part 2, 286–298, 1951.

In certain respects this paper generalizes the results of G. Krall [AMR **2**, Rev. 469]. Author treats a beam of constant flexural rigidity, subject to axial thrust and arbitrary lateral load, taking account of hereditary phenomena. The "memory" of the beam material is represented by a functional, whose properties are strictly given so as to secure existence and uniqueness of solution, which is represented by means of Green's function for a beam in the nonhereditary case. Folke K. G. Odqvist, Sweden

Plates, Disks, Shells, Membranes

(See also Revs. 1929, 1974, 1983, 2038)

1987. Martin, H. C., and Gursahaney, H. J., On the deflection of swept cantilevered surfaces, *J. aero. Sci.* **18**, 12, 805–812, Dec. 1951.

Authors present experimental evidence which shows existence of an apparent elastic axis for swept rectangular and triangular plates of uniform thickness. Deflections and slopes calculated from this axis show excellent and good agreement, respectively, with independent experimental data for high aspect ratios. Stresses are not expected to be determined from this method. Reviewer notes that recent NACA deflection data indicate the apparent elastic axis is a function not only of sweep angle but also depends strongly on aspect ratio for the lower ratios. Point should be further investigated.

Certain applications to finding natural frequencies of swept rectangular plates are also included.

Max L. Williams, Jr., USA

1988. Deverall, L. I., and Thorne, C. J., Bending of thin ring-sector plates, *J. appl. Mech.* **18**, 4, 359–363, Dec. 1951.

Authors attack the problem by a method they used for rectangular plates in a previous paper [AMR **4**, Rev. 3837]. General expressions for the deflection of plates whose planform is a sector of a circular ring are given for cases in which the straight edges have arbitrary but given deflection and bending moment. The solutions are given for all combinations of physically important edge conditions on the two circular edges. Sectors of circular plates are included as special cases. Solutions are given for a general load which is a continuous function of r and a sectionally continuous function of θ, where r and θ are the usual polar coordinates with the pole at the center of the ring. Several specific examples for angles of the sector 30° and 90° are given.

From authors' summary by George Z. Zizicas, USA

1989. Kokkinopoulos, E., Bending of thin plates (in Greek), *Tech. Chronika, Athens* **28**, 327/328, 286–295, Sept./Oct. 1951.

The last of a series of excellent articles, taken from a dissertation, on this important subject is presented. For the first article see AMR **3**, Rev. 241. Dimitri Kececioglu, USA

1990. Craemer, H., Some iteration and relaxation methods for cylindrical shells loaded symmetrically about its axis (in German), *Öst. Ing.-Arch.* **6**, 1, 35–42, 1951.

Author develops an iteration method for cylindrical shells which could be used for unusual load conditions; in standard conditions, author admits, the existence of tables or graphics gives easier results for tanks walls; e.g., Lewe's abacus [*Handbuch f. Eisenbetonbau* **9**, 4th ed., pp. 210–211].

Starting from the known fourth-order homogeneous differential equation, with exterior action on edges only, general solution is

given for deflections, slopes, bending moments, shearing, and normal forces. Then, for the same magnitudes, simplified values are given for the case of infinite slenderness with fixed conditions in the lower edge, be they actions (bending moments or shearing forces) or deformations (deflections, slopes). Determination of integration constants for a case of load is analyzed, using the equations for finite slenderness and by making a rapid iteration.

The direct determination of the moment and shearing forces in the lower edge by author's method is then analyzed; supposing infinite slenderness, it consists of two steps: Stresses in the lower edge are found, conditions in the upper edge not being fulfilled; next, the perturbations in the upper edge are removed, and so on. Applications are given for tank walls with (1) partial filling, (2) elastic fixing in the lower edge.

For short shells, an iteration method is proposed with polynomic series, an example being given.

Arturo M. Guzmán, Argentina

1991. **Seide, P., Compressive buckling of flat rectangular metalite type sandwich plates with simply supported loaded edges and clamped unloaded edges,** *NACA TN* 2637, 27 pp., Feb. 1952.

This is a revised version of a previous paper [AMR **3**, Rev. 1252]. In the theoretical development of the compressive buckling criterion, author incorporates the suggestion by Bijlaard that the vertical shear forces be carried not only by the sandwich core, but by the face material as well. Calculations by the present improved theory lead to slightly better agreement with experimental results than those by means of previous theory.

Theodore H. H. Pian, USA

1992. **Yen, K. T., Gunturkun, S., and Pohle, F. V., Deflections of a simply supported rectangular sandwich plate subjected to transverse loads,** *NACA TN* 2581, 39 pp., Dec. 1951.

Authors present charts for the maximum deflections of square sandwich plates subjected to uniform load over the entire plate surface or to a concentrated load at the plate center. The type of sandwich plate considered is one with isotropic faces and low-density, nonstress-carrying core. The calculations are made from equations based on sandwich-plate differential equations derived in *NACA TN* 2250 [AMR **4**, Rev. 2602]. Considerable mathematical manipulation of infinite series encountered in the equations for the deflections of plates subjected to a concentrated load at the plate center is presented in an appendix. These manipulations increase considerably the convergence of the series and facilitate computations.

It should be pointed out that the theory upon which the present paper is based contains an implicit assumption to the effect that the core is attached to the middle surface of the face sheets. In *NACA TN* 2637 (see preceding review) it is shown that this assumption may lead to results somewhat in error for the case of compressive buckling of sandwich plates with simply supported loaded edges and clamped unloaded edges. Whether errors of similar magnitude are obtained in the present problem is difficult to establish without recomputing a considerable number of the results from corrected equations. It is the reviewer's opinion that any errors are likely to be small.

Paul Seide, USA

1993. **Klitchieff, J. M., On the stability of plates reinforced by longitudinal ribs,** *J. appl. Mech.* **18**, 4, 364–366, Dec. 1951.

Author restates results given in a previous paper [AMR **2**, Rev. 854], modified for ease of direct determination of the stiffness of equispaced ribs required for specified critical compressive force on simply supported plate.

M. V. Barton, USA

1994. **Morgan, A. J. A., Uniformly loaded semi-infinite wedge-shaped plates,** *J. aero. Sci.* **18**, 12, 845–847, Dec. 1951.

A solution is found for wedge-shaped plates by suitable substitution for the deflection shape. This substitution defines the deflection along any radial line drawn through the apex of the wedge as the product of a function of the enclosed angle at the apex and the fourth power of the projection of the radial coordinate on the axis of symmetry. For the special case of semi-inifinite wedge plate with one edge free and one edge clamped under uniform load, it is found that all stress components are functions only of the square of the projected radial coordinate along any radial line through the apex. Possibility of extension to cover finite plate is indicated without too much justification.

Conrad C. Wan, USA

1995. **Winslow, A. M., Differentiation of Fourier series in stress solutions for rectangular plates,** *Quart. J. Mech. appl. Math.* **4**, part 4, 449–460, Dec. 1951.

Author discusses necessary mathematical conditions which should be satisfied in obtaining stress solutions for rectangular plates which involve the term-by-term differentiation of a Fourier series. Illustrative examples discussed are the rectangular plate with parabolic load distribution on two opposite edges and the uniformly loaded rectangular plate with built-in edges. It is concluded that, unless proved otherwise, mathematical requirements of termwise differentiation present additional restrictions on solutions which must satisfy given boundary-loading conditions.

A. M. Wahl, USA

1996. **Garcia Monge, F., Circular plates** (in Spanish), *Inst. tech. Constr. Cem.* no. 105, 91 pp., charts and examples, Jan. 1951.

Three general cases involving circular plates are considered, as follows: (1) Plates of constant thickness, symmetrically loaded; (2) plates of constant thickness, unsymmetrically loaded; (3) symmetrically haunched plates, symmetrically loaded.

The report is divided into two parts. In part I, the theory is presented and expressions are developed for use in solving various problems in each of the three general cases considered. The material contained in the first two cases is generally available, for example, in Timoshenko's "Plates and shells." In the third case, i.e., that in which symmetrically haunched plates are considered, the flexural stiffness $D = Eh^3/12\ (1 - \mu^2)$ is, of course, no longer constant. Several variations of the thickness h with respect to the radial distance from the center of the plate are studied, and expressions for the radial and tangential moments are obtained for several loading conditions.

In part 2, charts are given for the solution of 44 specific cases. Other solutions can be obtained by combination. All charts have been constructed for a Poisson ratio of 0.15, i.e., for use in reinforced-concrete construction. Seven examples are included to illustrate the use of these charts, and placement diagrams for the reinforcing steel are shown for each example.

James P. Michalos, USA

Buckling Problems

(See also Revs. 1991, 1993, 2011)

1997. **Kohler, K., Calculation of buckling of rods under a transverse load by means of the energy theorem** (in German), *Stahlbau* **20**, 12, 151–152, Dec. 1951.

Neglecting warping rigidity, author develops formulas for the critical load at the free end of a cantilever beam. The formulas are well known, but the article is of interest because of its way of handling the energy theorem. Reviewer prefers to state it as

follows: The vertical displacement of the point of application of a concentrated load has a critical value by which the strain energy in case of pure bending and in case of buckling becomes equal. The load causing such displacement is the critical load.

Sverre E. Kindem, Norway

1998. Brisby, M. D. J., The analysis of stiff columns, *Civ. Engng. Lond.* **46**, 546, 926–928, Dec. 1951.

The unsymmetrical bending formulas for composite prismatic beams loaded in axial eccentricity are presented in three figures and nine equations. Two additional figures illustrate graphically how the radii of the ellipse of gyration may be considered the mean altitude of a triangle whose vertexes are the neutral axis and eccentric load coordinates. This follows from the antipolar properties of an ellipse and results in a reciprocal relation of the neutral axis.

Four examples illustrate kernel and kern-distance determinations, though, regrettably, not graphically as much mathematical elaboration exists already [Cross and Morgan, "Continuous frames of reinforced concrete," p. 54]. Reviewer cautions against analytic continuation of the tangent into the interior of the ellipse as existing only in an imaginary plane. The graphical neutral axis determination is somewhat of an extension of the formulas given in Southwell's "Theory of elasticity," paragraphs 172–4 and example 15.

Rudolph L. Leutzinger, USA

1999. Silver, G., Critical loads on variable-section columns in the elastic range, *J. appl. Mech.* **18**, 4, 414–420, Dec. 1951.

This simple and practical method for determining critical loads of variable-section columns illustrates the well-known classical approach given by L. Vianello in 1898, and the method used for numerical integration has much in common with that of N. M. Newmark [*Trans. ASCE* **108**, p. 1161, 1942]. No reference is given to these authors or to any other previous solution of such problems. It is shown that satisfactory results are obtained if, for the tentative equation of the buckling curve, the simplest polynomial is used which will satisfy the end conditions and follow the known general shape of the curve. A closer, second approximation can be obtained in the usual manner by using for it the results obtained in the first approximation.

George Winter, USA

2000. Bogunović, V., Buckling of coverplates of rib structures (in German), *Publ. math. Inst. math. Acad. Serbe Sci.* **3**, 271–286, 1950.

Author considers a long plate of finite width reinforced by equally spaced ribs perpendicular to its length. Structure is loaded by lines of force uniform along the width and in the plane of the ribs. Assumption is made that state of stress in plate and ribs is plane. On this basis, author computes critical load for a particular combination of dimensions of rib and plate, assuming that the cover plate buckles into a half wave between two ribs. With these results, author computes effective width for substitute plate. Some typographical errors occur.

Marvin Stippes, USA

2001. Stüssi, F., Kollbrunner, C. F., and Walt, M., Report of tests on buckling of Avional M plates, hard annealed, under unilateral uniformly and nonuniformly distributed force (in German), *Mitt. Inst. Baustatik ETH, Zurich* no. 25, 156 pp., 1951.

Experiments investigated elastic buckling of rectangular plates, uniformly and nonuniformly compressed in one direction. Loaded edges were simply supported, the unloaded edges having various combinations of simply supported, clamped, and free. Plates tested had lengths of 48 cm and 36 cm, widths of 12, 16, 20 cm, and thickness of 1, 2, 3, 4, 5, and 6 mm. Special testing machine using dead weights was used, and lateral deflections at various points of the plate were measured. Loads were increased above buckling, and deflection measurements enabled detailed wave pattern to be plotted.

For the simply supported plates, good agreement was obtained between experimental and theoretical values of $k_{\min}$. However, for the cases with clamped edges, the experimental values were consistently lower than the theoretical values, due to the inability to provide absolute rigidity of the supports. The experimental values then corresponded to elastically clamped plates.

Reviewer was impressed by the elaborate precision of the test set-up, which clearly demonstrated the buckling phenomenon in plates. In contrast to previous tests, initial curvatures in the plate had little effect either on the buckling load or ultimate load.

Harold Lurie, USA

2002. Kollbrunner, C. F., and Herrmann, G., Effect of Poisson's ratio on the stability of rectangular plates (in German), *Mitt. T.K.V.S.B.*, Verlag Lehmann, Zürich, no. 4, 20 pp.

From well-known theories, authors compute numerically the influence of Poisson's ratio ν on buckling loads of rectangular plates in the elastic domain. Diagrams of buckling load are presented for different boundary and loading conditions for different values of ν between the theoretical limits $0 \leqslant \nu \leqslant 0.5$.

It is established that the buckling load is strongly dependent on ν if one edge of the plate is free.

Frithiof I. Niordson, Sweden

2003. Flint, A. R., The lateral stability of unrestrained beams, *Engineering* **173**, 4486, 4487; 65–67, 99–102, Jan. 1952.

In order to check the theory of lateral stability of beams under the action of vertical point loads, a series of tests on xylonite models and small steel and light-alloy beams have been carried out. Test rigs and load hangers were constructed capable of simulating the idealized conditions which are assumed as a basis for theoretical solutions (e.g., free bending and axial, but no torsional movement at the supports; the load being free to move sideways with the beam, its line of action remaining vertical). Experiments were made with beams of I-, T-, and channel sections under point loads at midspan or at some noncentral point along the span, the load being applied at the shear center as well as above or below it. Conditions for instability were sharply defined, and buckling loads could be observed with only about a 2% possible error.

When the influence of the warping restraint of the cross section during torsion is included in estimates of buckling stresses, agreement between estimated and observed values was satisfactory in all cases (about 5% error). Formulas for estimating critical loads are not derived in this paper although they seem to have been developed originally by author in many special cases. Design rules proposed in B. S. Code of Practice, based on results for members under uniform bending moment, have been proved to be extremely conservative when applied to cases of point loading, and should be replaced by author's formulas based upon theoretical solutions, wherever detailed design is necessary.

H. Kauderer, Germany

2004. Funk, P., On a stability problem occurring in an arched measuring tape, the hollow side outwards (in German), *Öst. Ing.-Arch.* **5**, 4, 387–397, 1951.

The discussion is limited to bending in one plane without twisting of a tape with a circular arc cross section. Buckling is sudden from the arched configuration to the locally flattened tape and is treated as a discontinuous process. A general discussion is given

of several extremal problems in analytical and geometrical terms, and the difference between neutral instability and the jump type is explained. Diagrams are drawn and numerical values given for strain energy and bending moment against curvature and also for critical points. D. C. Drucker, USA

2005. **Klein, B., Parameters predicting failure of partial diagonal tension beams,** *J. aero. Sci.* **19,** 2, 141–142, Feb. 1952.

On empirical grounds, author chooses dimensionless parameters for correlating data on web ruptures and stiffener failures of diagonal-tension beams of the type used in aircraft. Test data from several sources provide reasonably smooth curves when the proposed parameters are used as coordinates.

H. L. Langhaar, USA

2006. **Lubinski, A., Buckling of rotary drilling strings,** Drill. Product. Practice, Amer. Petr. Inst., 178–214, 1951.

Corrected and completed paper reviewed in AMR **5,** Rev. 399.

Joints and Joining Methods

2007. **Flanigan, A. E., and Kaufman, M., Microcracks and the low-temperature cooling rate embrittlement of welds,** *Weld. Res. Suppl.* **16,** 12, 613s–622s, Dec. 1951.

Article deals with the effect of microcracks on fatigue, tensile, and notch-impact tests. Microcracks caused a 40% reduction in fatigue strength. It was found that the cracks were most prominent in an axial plane that extends parallel to the surface of the plate, and that these cracks may be best detected by electrolytic polishing. Detection of the cracks is difficult since they do not extend to the surface of the weld. The tests indicated that hydrogen and fast cooling are important factors in the formation of microcracks. Glen N. Cox, USA

Structures

(See also Revs. 1987, 1990, 1994, 1996, 2039, 2208)

2008. **Raczat, G., Elimination of iteration in the Cross method** (in German), *Bauingenieur* **27,** 2, 49–52, Feb. 1952.

As author states, method presented is very similar to one proposed by Lin in 1934 and one which has not received due consideration. Nevertheless, principal formulas are obtained differently and procedure is more mechanical.

Method consists essentially in calculating "real" stiffness and carry-over factors in systems without sidesway, based on convergent geometrical series. Simplicity is obtained by assuming that subsequent joints do not rotate. Sidesway is taken care of in a second step, similar to that introduced by Morris.

In reviewers' opinion, method could be presented as the "calculation of the conjugate matrix," which is formed by the real stiffness and carry-over factors of a system. Once this conjugate matrix is known, the system can be solved easily for any number of different cases of loading.

Doubtless a valuable contribution to the resolution of continuous frames. A. J. Bignoli and E. Rathgeb, Argentina

2009. **Michalos, J. P., Analysis of skewed rigid frames and arches,** *J. Amer. Concr. Inst.* **23,** 6, 437–454, Feb. 1952.

Paper examines single-span frames and arches of any shape under loads of any direction. As, in the general case, six geometrical conditions have to be satisfied, solution is simplified by introducing centroidal axes depending on the distribution of the various kinds of stiffnesses along the arch. There are nine pages of numerical examples. The distribution of moments and shears over cross section is consciously disregarded, i.e., analysis holds for infinitely small width of arch. Reviewer thinks, however, that, under this assumption, the idea of skewness has no meaning for a completely clamped arch loaded in its plane.

H. Craemer, Germany-Egypt

2010. **Ughi, L., On the calculation of the combined system of arch and beam** (in Italian), *Atti Ist. Sci. Costr. Univ. Pisa, Pubbl.* 19, 24 pp., 1951.

Accepting a few very reasonable hypotheses that simplify the problem, author calculates a symmetrical arch taking into account the action of superstructure. The representative system is formed of a biarticulated arch of straight elements and a beam, joined with pendular elements.

Author chooses the unknowns (bending moments in the sections where pendular elements join the arch and arch thrust) so as to obtain a set of equations of three terms and a fourth one due to the thrust. Besides the advantage of giving equations in which many coefficients vanish and the diagonal ones predominate, the bending moments in the arch are generally small quantities. Problem's simplification allows author to tabulate values to obtain the coefficients for the set of equations.

It is reviewer's opinion that this work is an excellent example of how a good choice of the fundamental system can make easier the solution of an apparently complex problem, with all the practical advantages this implies. Arturo J. Bignoli, Argentina

2011. **March, H. W., Sandwich construction in the elastic range,** *Symp. Struct. Sandwich Constr. ASTM Spec. tech. Publ.* 118, 32–45, 1952. $2.

A brief survey of a number of problems covering sandwich construction in the elastic range is presented with formulas and curves given for constructions having isotropic facings and cores, although the actual results, obtained at the Forest Products Laboratory, were for orthotropic materials. Some 32 references are listed in the accompanying bibliography.

Author discusses the elastic properties of facing and core materials, wrinkling of the facings, concentrated and uniform loadings of a sandwich beam, uniformly loaded flat rectangular sandwich panel, stability of a flat rectangular sandwich panel under compressive end loads for different edge conditions, and the stability of flat rectangular sandwich panels in shear.

Frederick K. Teichman, USA

2012. **Ruffner, B. F., and Hout, E., Stresses and deformations in wings subject to torsion,** *NACA TN* 2600, 79 pp., Feb. 1952.

Approximate solution for stresses and deformations in a thin-walled cylindrical box, with continuous rigid framing and one end section clamped, subject to constant torque. Wall thickness is uniform. Contour shape is represented by Fourier series of the coordinate along periphery. Analogous expansion is adopted for warping stresses; component amplitudes are supposed to be exponential functions of axial coordinate. Twist and stresses are calculated for rectangular, elliptical, and airfoil cross section.

P. Cicala, Argentina

2013. **Walls, J. H., Investigation of the air-compression process during drop tests of an oleo-pneumatic landing gear,** *NACA TN* 2477, 17 pp., Sept. 1951.

Paper presents experimental study to determine type of air-compression process and its effect on loads during landing impact for a small oleo-pneumatic landing gear. Analysis indicates air-compression exponent value should have relatively little effect on

loads through most of impact, but due to differences at end, may affect total load, depending on reduction in maximum strut stroke caused by increases in polytropic exponent. For these tests, polytropic exponent ranged between 1.01 and 1.1 and appeared practically independent of vertical velocity. Value of 1.06 appeared an adequate average. Curves are presented describing load-stroke, pressure-stroke, stroke time, and exponent-contact velocity relationships, but appear to have application only to tested strut.

Stanley I. Weiss, USA

2014. Raphael, J. M., The development of stresses in Shasta dam, *Proc. Amer. Soc. civ. Engrs.* **78,** Separ. no. 117, 21 pp., Feb. 1952.

With the objective of giving continued assurance of structural safety and to furnish data leading to refinements in design, the U. S. Bureau of Reclamation has made, since 1926, detailed measurements of the structural behavior of its major dams. Paper is limited to a presentation of the stresses found at the base of the spillway section of Shasta Dam. Details are given of construction and layout of the strain gages used. As the deformations measured are due to many sources, author analyzes each one of the different causes of volume changes and shows that, in the case of Shasta Dam, only stresses, temperature, and creep have had sensible influence on the strains. Temperature effect is easily determined and, to eliminate the deformation due to creep, a very simple and effective computation procedure is explained. Using this process, stresses were computed at each point of measurement for each direction covered by the strain meters, and paper shows in several graphs the distribution, along the section studied, of normal stresses in vertical and horizontal directions (parallel and normal to dam axis) and of shear stresses, at various stages of construction and reservoir heights for the years 1943–1946. The distribution of vertical normal stresses disagrees entirely with the usual assumption that stresses increase from low compression at the upstream face to higher compression near the downstream face. A balance was made between external loading (water and weight of concrete) and the measured stresses, resulting in a satisfactory check of the two conditions of equilibrium at the time of maximum water loading, considering the dam wall as a cantilever, corroborating the designers' assumption that the arching of Shasta Dam is too slight to be of appreciable structural value. Author shows that the unaccustomed appearance of the vertical stress-distribution diagrams cannot be explained only by temperature conditions nor by difference between rigidity of foundation rock and dam concrete; the reason lies principally in the method of construction. The most interesting conclusion is that, at Shasta Dam, construction techniques and temperature change have a greater influence on stresses than water load.

Ivo Wolff, Brazil

2015. Chow, V. T., Hydrodynamic pressure due to horizontal earthquake shock computed by curves, *Civ. Engng., N. Y.* **21,** 9, 52–53, Sept. 1951.

Curves are given based on Westergaard's theoretical study of hydrodynamic forces on dams due to earthquakes [*Trans. ASCE* **98,** p. 418, 1933]. M. P. White, USA

2016. Symonds, P. S., and Neal, B. G., Recent progress in the plastic methods of structural analysis, *J. Franklin Inst.* **252,** 5, 6; 383–407, 469–492; Nov., Dec. 1951.

Paper is a detailed account of the present status of structural analysis by plastic methods. First sections state the basic hypotheses. Next few sections cover details of the preliminary steps necessary to a complete analysis. Then comes a statement of the Greenberg-Prager theorems which are the basis of the method. Sections 8–10 are concerned with several examples of proportional loading which are by no means trivial. Several methods for finding upper and lower bounds for the collapse load are mentioned, and the authors' original method of "elementary mechanisms" is described in some detail. In Section 11, new results on estimation of deflections are presented. While results are admittedly less accurate than load estimations, two primary sources of error are shown to have opposite effects, thus frequently leading to good results. This section is particularly valuable since it represents a major step in overcoming the primary drawback of limit analysis to date. The final sections extend previous results to more general types of loading.

Paper is competently written, and is definitely directed toward engineers. Mathematical proofs of the theorems are replaced by convincing heuristic arguments (but references are given for rigorous proofs). The aim throughout is to find methods suitable for solving practical problems rather than results which are mathematically elegant. The "elementary mechanisms" technique is a prime example of this approach, since the experience and intuition of the engineer are utilized in applying it.

Authors' appraisal of their results is to be commended. Basic assumptions, advantages and disadvantages, limitations as applied to actual materials, and comparison with experimental results are clearly and completely stated. A list of 38 references is frequently and specifically referred to.

Philip G. Hodge, Jr., USA

2017. Johnson, A. W., Frost action in roads and airfields. A review of the literature 1765–1951, *Nat. Res. Counc. Highway Res. Bd. Publ.* 211, 287 pp., 1952. $3.

2018. Giles, C. G., Skidding and the slippery road, *Trans. Instn. Engrs. Shipb. Scot.* **95,** part 4, 195–232, 1951–52.

Paper gives the results of investigations into the effect of vehicle, tire, and road surface characteristics on skidding. Methods of measuring the friction between tire and road are described and results obtained under different conditions are given. The results show that there are big differences in the performance of wet roads, and a desirable standard of performance is considered in the light of tests on accident sites and on some present-day roads.

The connection between types of road construction, road-surface texture, tire-tread pattern and skidding resistance on wet roads is discussed, and suggestions are made as to the way motorists and others can help to eliminate skidding accidents.

From author's summary

2019. Campus, F., Prestressed concrete (in French), *Ann. Trav. publics Belg.* **103,** 1, 2; 19–47, 295–331; Feb., Mar. 1950.

Paper is a very detailed report on the tests of various types of prestressed concrete members. Beams of the type HOYER (tee, with embedded thin wire, reinforcement), LESAGE (rectangular, with embedded thin wires), and WETS (rectangular, variable cross section, exposed wires) were tested in bending. Prestressed concrete pipes, both with prestressed longitudinal and spiral reinforcement were tested in bending, diametral compression, and internal pressure. Finally, railroad ties of prestressed concrete were tested both under static and cyclic loads. Very elaborate experiments were directed toward the investigation of elastic behavior, the phenomena of cracking and failure.

Author's conclusion brings out the following points: (1) Both in the case of beams and of tubular members, the general behavior follows the laws of elasticity, well beyond the first cracking and in most cases up to the failure, which often is of a sudden type. (2) It is very difficult to measure directly the tension in the reinforcement. Different indirect methods were tried out. (3) The

evaluation of the effect of shrinkage and plastic deformation of the concrete and its influence on the prestress remains a problem to be studied further. (4) The failure under static loads is due mainly to the crushing of the concrete; under cyclic loads the slippage or even the rupture of the reinforcements occurs. (5) The longitudinal prestressed reinforcement in tubular section is very effective in increasing the flexural rigidity. The prestressed spirals increased the resistance under diagonal and internal pressure. (6) The proper evaluation of the factor of safety is still a very complex problem which requires more experience.

D. Vasarhelyi, USA

2020. **Manning, G. P., Analysis of groups of piles by the displacement method,** *Concr. Constr. Engng.* **47,** 1, 3–7, Jan. 1952.

Each pile in a group is assumed fixed at its tip and monolithic with other piles or the foundation at its upper end. Problems are solved by a standard procedure of analysis using joint rotation and displacement. Effect of variation of soil strata on the bending moments is considered. Elio D'Appolonia, USA

2021. **Jakowlew-Herbaczewski, P., Concrete footings for walls and columns,** *J. Amer. Concr. Inst.* **23,** 4, 333–351, Dec. 1951.

Equations are developed from which charts and tables are prepared which permit the determination of optimum dimensions for concrete footings. The effect of varying costs of materials is considered, and a method is given for determining the economical percentage of steel in concrete.

The aim of the equations, charts, and tables is not only to aid the designer in saving time and tedious labor of "cut and try" methods, but to present a clear and broad view of the whole range of possibilities in dimensions and proportions open to the judgment and choice of the designer in figuring wall or column footings. No new theory is presented, but from commonly known equations and relations, graphs have been prepared representing the necessary data for a rapid selection of the best footing for the job at hand. The data and curves for diameters of bars as well as equations for economical percentage of reinforcement are perhaps novel. Their demonstrated usefulness recommends their wider application by designers.

From author's summary by R. E. Fadum, USA

2022. **Murray, J. M., Main longitudinal stresses in ships,** *Engineering* **172,** 4472, 469–472, Oct. 1951.

Author surveys development of classical method of determining longitudinal strength of ships. Survey notes key contributions from Bouguer's paper of 1746 to present-day papers. Of interest is the present trend to examine the bending moment in its two components, that due to still-water bending, and that due to wave action. Author supports this trend and reports results of recent studies of wave action and of wave characteristics. Appendix of article gives derivation of variation of bending moment with length of wave. Author recommends that magnitude of bending moment for ship in still water be determined by statics; or, bending moment = moment-due-to-weight minus moment-due-to-buoyancy. Moment values may be expressed in terms of geometrical properties of hull and the block coefficient. Moment due to waves is computed separately. Values of corrective coefficients vs. block coefficients are given in appendix. Author states that: "Most direct method of comparing, in similar ships, stresses induced in service is to consider the stresses calculated in the still-water condition." Cameron M. Smith, USA

2023. **Lelli, M., Precompression and state of tensions after setting, in lined pressure galleries** (in Italian), *Energia elett.* **29,** 2, 69–72, Feb. 1952.

Theoretical analysis of the effect of precompression in the lining of a tunnel attained by injecting cement mortar (pressured grouting) between rock and lining. Due to shrinkage and plastic flow of the grouted interlayer, sensible decompression will occur which is theoretically investigated in this paper under the assumption that, after setting, the elasticity of hydraulic contraction does not exist. Reference is made to the study of Ugo Carpino determining the pressure upon a tunnel lining due to cement mortar injected under pressure between the rock and extrados of the lining, assuming certain compressibility after setting and due to effect of shrinkage. Numerical examples show that the decompression amounts to up to 7%. J. J. Polivka, USA

📖2024. **Zanoskar, W., Construction of galleries and tunnels [Stollen- und Tunnelbau. Eine Einführung in die Praxis des modernen Felshohlbaues],** Wien, Springer-Verlag, 1950, x + 231 pp., 74 figs. $5.80.

Author treats tunnel construction from the viewpoint of the mine superintendent and shift foremen. He has compiled a brief but comprehensive summary of current German methods of driving tunnels, including drilling, blasting, and mucking procedures. Method of computing progress is given and ventilation requirements are discussed. Power requirements and costs are also taken up. The first part of the book is devoted to sound-rock tunneling while the latter part takes up tunnel work in loose rock. A short bibliography of German articles on tunnel construction is included. J. R. Bruggeman, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 1972, 2048, 2057, 2075, 2076)

2025. **Bollenrath, F., and Troost, A., Relations between the stress and deformation gradient. II. Influence of shape and scale** (in German), *Arch. Eisenhüttenw.* 22, 10, 327–335, Sept./Oct. 1951.

Methods of AMR **4,** Rev. 2032 are applied to bending and torsion of cylinders of various symmetrical sections. Analogous methods are developed to analyze the dependence of fatigue strength on scale, and are applied to experimental data.

F. R. N. Nabarro, England

2026. **Rozovskiĭ, M. I., Flow and prolonged failure of materials** (in Russian), *Zh. tekh. Fiz.* **21,** 11, 1311–1318, Nov. 1951.

An integral equation was proposed as a result of an analysis of flow curves made in conjunction with equations suitable to investigations of flow and relaxation processes. This integral equation provides stress, strain, time relationship. Dependence of the relaxation time upon stress is taken into account. Further, this equation was employed as a basic relationship for an investigation of the kinetics of crack propagation in various materials. A relationship between instantaneous and prolonged strengths for aging materials was established.

From author's summary by V. A. Valey, USA

2027. **Johnson, A. E., Creep under complex stress systems at elevated temperatures,** *Instn. mech. Engrs.* **164,** 4, 432–446, Proc. 1951.

Paper discusses salient points of author's 150-hour tension-torsion creep tests of metals—a 0.17% C steel at 350 C [*Engineer* **188,** p. 126], an aluminum alloy at 150 C and 200 C [AMR **3,** Rev. 2669], a magnesium alloy at 20 C and 50 C [AMR **4,** Rev. 1155], and a nickel-chromium alloy at 650 C (Appendix 1).

It is found that for all metals and temperatures, the creep rates in terms of octahedral shear strain of a specimen under various

combined stresses are functions of the applied octahedral shear stresses; for isotropic metals subjected to moderate stresses, the principal creep rates could be well represented by equations of the Saint Venant-Mises type

$$C_1 = A\ [\Sigma\ (\sigma_1 - \sigma_2)^2]^{p_1}\ [(\sigma_1 - \sigma_2) - (\sigma_3 - \sigma_1)], \text{ etc.}$$

where σ is the principal stress, and A and p are constants. Author claims that the plots of Lode's variables of stresses and creep rates confirm the above statements. For metals having directional properties under high stresses, the principal creep rates are represented by equations similar to the above with additional coefficients of the following form

$$C_1 = \{A_1\ [\Sigma\ (\sigma_1 - \sigma_2)^2]^{p_1} + A_2\ [\Sigma\ (\sigma_1 - \sigma_2)^2]^{p_2} + \ldots\}$$
$$\{C_1\ (\sigma_1 - \sigma_2) - B_1\ (\sigma_3 - \sigma_1)\}, \text{ etc.}$$

where A, p, C_1, and B_1 are material constants and depend on stress states. In the test results reported, the creep strains under consideration were so small that the variation of stresses due to deformations was negligible. From experience, reviewer believes that the problem will be more involved once the deformation becomes comparatively larger.

In appendix 1, the 150-hour tension-torsion creep test of a nickel-chromium is given. In appendix 2, author attempts to study the supposition that the hydrostatic pressure does not have any modification on the creep-rate state for a metal subjected to combined stresses. The tension-tension creep tests of a magnesium alloy are given; their comparison with the tension-torsion creep tests gives some support to the supposition. Reviewer believes that this is an important advance in creep study. In Appendix 3, author shows several possible applications of the Saint Venant-Mises-type creep-rate equations, such as a thin-walled tube subjected to internal pressure and axial loads, or to internal pressure and pure bending, and a hollow cylinder subjected to internal pressure. However, reviewer feels that the redistribution of stresses due to creep and the theory of plasticity should be considered in applications having nonuniform stress distributions.

Ling-Wen Hu, USA

2028. Wood, W. A., Wilms, G. R., and Rachinger, W. A., Three basic stages in the mechanism of deformation of metals at different temperatures and strain-rates, *J. Inst. Metals* **79**, part 3, 159–172, May 1951.

Deformation of aluminum is accompanied by changes in microstructure, depending on rate of strain and temperature. Three mechanisms are found: Slip, cell mechanism, and boundary microflow. Present paper is a study of the latter. The cells, into which the grains dissociate at high temperature and low values of strain, eventually become comparable in size with the grain itself. Then the third mechanism sets in, in which the grain may deform entirely by micromovements. Results indicate a concentration of such movements at grain boundaries. Analysis provides for satisfactory understanding of strength and strain-hardening, and creep.

B. Gross, Brazil

2029. Lücke, K., and Lange, H., On the form of the strain-hardening curve of purest aluminum crystals and the formation of deformation bands (in German), *Z. Metallk.* **43**, 2, 55–66, Feb. 1952.

The strain-hardening curves of single metallic crystals, according to previous investigators, fall into two groups: Single metallic crystals having a face-centered regular lattice deform according to an ordinary parabola, and those of the hexagonal system according to a straight line law if the shearing stress in the plane of slip and in the direction of slip is plotted vs. the unit shearing strain. However, systematic deviations from the parabolic law have been observed by several investigators, particularly by R. W. Cahn, in high-purity aluminum single crystals. Authors devote their paper to these deviations by comparing the flow curves for various orientations of single crystals for a 99.5% aluminum with those for a high-purity aluminum of 99.99%. They found that, for the latter, double slip may start at a certain strain, a fact already known. Whereas the 99.5% Al crystals disclose a very dense system of slip lines, the 99.99% Al had a course system of slip planes, and these planes were always intersected by "deformation bands" not coinciding with any plane having a rational index of orientation relative to the crystal lattice. The deformation bands appear at a later stage of the stretching in a steeper portion of the strain-hardening curve, and in them a bending of the slip planes becomes distinctly visible. The formation of the course parallel system of these deformation bands is probably due to an instability phenomenon—the regular, simultaneous slip in the crystallographic slip planes stops. The deformation bands do not seem to form simultaneously but appear one after the other, successively.

A. Nadai, USA

2030. Johnson, A. E., Jr., and Batdorf, S. B., A study of slip formation in polycrystalline aluminum, *NACA TN* 2576, 18 pp., Dec. 1951.

Paper analyzes the assumptions made by Sachs, Cox and Sopwith, Taylor, and Batdorf and Budiansky in attempts to correlate mathematical and physical theories of plasticity. 2S-0 aluminum alloy was tested in tension and X350 photomicrographs of this specimen at various strains were studied. Observations show: (1) Slip occurs first in isolated grains. (2) As straining increases, there is a tendency for interaction among those grains which first suffered plastic deformation and their neighboring grains. (3) Slip occurs on only one slip plane within a grain. (4) Microscopic and macroscopic strains are not identical. (5) Microscopic and macroscopic stresses are not identical. (6) Results are in better agreement, though, with the assumption that microscopic and macroscopic stresses are identical than with the assumption that respective strains are identical. (7) The resolved shear stress is not equal in all grains. (8) The experiments are compatible with, but do not necessarily verify the assumption that plastic deformation in strain-hardening materials is primarily due to slip within the grains, because slip was first observed on the photomicrographs in which the stress was considerably beyond the elastic limit.

Dimitri Kececioglu, USA

2031. Hedgepeth, J. M., Batdorf, S. B., and Sanders, J. L., Jr., On the angular distribution of slip lines in polycrystalline aluminum alloy, *NACA TN* 2577, 18 pp., Dec. 1951.

A photomicrograph of polycrystalline aluminum alloy deformed in simple tension (see preceding review) is analyzed to give statistical distribution of angle between tensile axis and trace of slip planes on a polished face. Comparison with theory of Batdorf and Budiansky [AMR 2, Rev. 1264] shows significant disagreement, probably because theory considers only one slip system, whereas aluminum has 12 systems. Reviewer notes that preferred orientation shown by x ray is rather strong.

F. R. N. Nabarro, England

2032. Brown, A. F., Slip bands and hardening processes in aluminium, *J. Inst. Metals* **80**, 115–124, 1951–1952.

Fine structure of slip bands on high-purity aluminum crystals is investigated by electron-microscope methods. At small strains, each slip band consists at all temperatures of only one glide lamella. With increasing strain, new bands are predominantly formed at low temperatures, but new lamellae arise in already existing bands at high temperatures. These results lead

to a simple picture on variation of internal stresses between glide lamellae which suggests that slip, by forming new bands, contributes much more to macroscopic strain-hardening than slip by forming new lamellae within existing bands. On this basis, stress-strain curve under different conditions of temperature and strain rate may be quantitatively interpreted. As a consequence, a mechanical equation of state cannot exist except at very small strains. Albert Kochendörfer, Germany

2033. **Opinsky, A. J., and Smoluchowski, R., The crystallographic aspect of slip in body-centered cubic single crystals. I. Theoretical considerations,** *J. appl. Phys.* **22,** 11, 1380–1384, Nov. 1951.

Present part deals with theoretical considerations as to what glide systems operate in body-centered cubic crystals and what their critical shear stresses are. Assuming the glide direction to be a [111] direction independent of composition and temperature (as indicated by all experiments), resolved shear stress is calculated in terms of Force and Miller indexes of the normal-to-slip-plane and force directions. This permits assigning the regions in unit stereographic triangle within which a given glide system has a greater stress than any other of those under consideration. The fact that ratio of resolved shear stresses for various slip planes remains constant along a great circle passing through the force and glide directions, enables the projection of points of unit triangle on one side of it, thus reducing the problem to one dimension. On this basis, an analysis of experimental results on iron-silicon alloys and a reinterpretation of data available from the literature will be given later in part II.

Albert Kochendörfer, Germany

2034. **Smakula, A., and Klein, M. W., Investigation of the gliding process in ionic crystals by prismatic punching,** *Phys. Rev.* (2) **84,** 5, 1043–1049, Dec. 1951.

Method involves application of a static force on a small prismatic punch with square or circular cross section ($\sim$0.1 mm^2) on surface of a thin crystal (optimum thickness $\sim$1 mm), lying on a softer support. The part under punch glides through crystal in glide direction along glide planes, thus causing one or more elevated mounds on opposite surface according to number and orientation of glide systems. For thallium bromoiodide crystals, method has given following results: Glide planes (110); glide directions [100]; glide bands, ranging from 1 to 5 microns, apart from each other 100 atoms and 15 atoms at room temperature and at -190 C, respectively; relation between stress σ and shear angle δ is linear; transient creep follows Andrade's law $\delta = at^n$. For sodium chloride, glide takes place along (110) planes in [110] direction. Apparent cleavage along dodecahedron planes and normal cleavage along cubic planes are explained as ruptures between neighboring glide planes.

Albert Kochendörfer, Germany

2035. **Benbow, J. J., The measurement of the viscosity of highly viscous plastic mixes at high rates of shear,** *Brit. J. appl. Phys.* **2,** 12, 359–363, Dec. 1951.

Paper describes construction and operation of a pendulum viscometer suitable for testing pastes with viscosities of several hundred poises at rates of shear from 5000 to 25,000 sec^{-1}. The instrument is constructed from a Goodbrand ballistic yarn tester by changing the pendulum and adding appropriate shearing surfaces. The sample is sheared between fixed surfaces at the bottom of the pendulum arc and parallel surfaces mounted on the sides of the pendulum.

Equations are given for calculating apparent Newtonian viscosity and rate of shear from pendulum mass and moment of inertia, dimensions of the instrument and height of pendulum rise after passing through the paste. Errors due to change in linear velocity of the pendulum on passing through the paste and the variation in velocity with radial distance from the point of suspension are shown to be small compared to the rise in temperature of the paste, which is of the order of 13 C at 25,000 sec^{-1}.

Data on polyvinyl chloride pastes are presented to show variation in apparent viscosity with rate of shear as well as thixotropy and strain-hardening effects. S. P. Jones, USA

2036. **Jacquesson, R., and Manenc, J., Displacements of crystalline particles in a monocrystalline aluminum plate during rolling** (in French), *Rev. Métall.* **48,** 11, 883–887, Nov. 1951.

Paper first discusses x-ray studies of torsional deformation of monocrystalline aluminum wire 1 mm in diameter. Analysis is made of geometrical transformations occurring during the rolling of isotropic sheet. Experiments are described to measure distortion by inserting inclusions in monocrystalline sheet, which is then correlated with geometrical analysis. One conclusion is that particles produced by progressive destruction of initial monocrystal become oriented following a simple law, as if metal were isotropic. Herbert I. Fusfeld, USA

2037. **Hasiguti, R. R., and Hirai, T., Internal friction of cold-worked single crystals of copper,** *J. appl. Phys.* **22,** 8, 1084–1085, Aug. 1951.

Effect of cold-working on the internal friction of copper single crystals was studied by measurements on a longitudinally vibrating specimen. Curves show low internal friction at very small and at large amounts of cold work, with a definite maximum in the range of cold-working equivalent to a resolved shear strain in the slip direction of 0.1. Reviewer notes that the form of the curves confirms the qualitative data of Read [*Trans. AIME* **143,** 30, 1941]. Observed decrease of internal friction of the cold-worked specimens with time at room temperatures seems to be contrary to Read's observations, and the numerical values of the internal friction of annealed specimens seem somewhat higher than those measured by Read. Donald E. Hudson, USA

2038. **Shevchenko, K. N., Axisymmetrical elastoplastic problem for a plate weakened by a circular hole** (in Russian), *Prikl. Mat. Mekh.* **15,** 4, 519–520, July-Aug. 1951.

This note contains a solution of plane elastoplastic problem for infinite thin plate axisymmetrically loaded on the boundary of a circular hole. Solution is obtained in closed form integrating a differential equation of Lagrangian type. In the plastic region, material is supposed to be incompressible, and work-hardening to follow linear law. Dragoš Radenković, Yugoslavia

2039. **Merlino, F. S., Introduction of new constraints after application of exterior loads. Viscous phenomena** (in Italian), *Atti. Accad. naz. Lincei R. C. Cl. Sci. Fis. Mat. Nat.* (8) **11,** 1/2, 58–60, July/Aug. 1951.

Author studies an arbitrary linear, homogeneous viscoelastic system with $n - 1$ redundancies ($n \geqslant 1$), and applies load at $t = 0$. As soon as system has taken initial deformation, a new constraint A is introduced, giving a system of n redundancies of which reaction X at A is zero for $t = 0$. But for viscoelastic behavior of material, X would remain zero for $t > 0$. Introducing a Volterra dislocation, corresponding to a suitable displacement at A for $t = 0$, author compares values of X with reaction X_0 at A, which would prevail if constraint A were introduced before application of load. Thus he finds again a relation by F. Levi [AMR **4,** Rev. 4454]. Case of several $(n - k)$ redundancies of type A is discussed. Folke K. G. Odqvist, Sweden

2040. **Koehler, J. S., The production of large tensile stresses by dislocations,** *Phys. Rev.* (2) **85,** 3, 480–481, Feb. 1952.

It is shown that large tensile stresses arise in a stressed material containing locked dislocations.

F. Schultz-Grunow, Germany

2041. **Peach, M. O., The concept of force in dislocation theory,** *J. appl. Phys.* **22,** 11, 1359–1364, Nov. 1951.

The concept of force acting on a dislocation is intrinsically different from the classical concept of force on masses, but can be defined in terms of energy changes of an entity whose position changes also. In statical and quasi-statical processes, forces of constraint and dissipation are defined in terms of this field force. Resultant force gives direction of actual movement when constraints allow it. In linear elasticity and for infinite crystals, the third law of Newton and the parallelogram law of addition are valid. After applying these laws to a crystal containing imperfections, author discusses thoroughly the literature relating to force.

Leonardo Villena, Spain

2042. **Mironoff, N., A new method of micromechanic testing of metals** (in French), *Rev. Soudure* **7,** 4, 214–232, 1951.

Author approaches the realization and the interpretation of bending tests for ductility determination from a new point of view. He suggests that the bars used as test specimens be bent under constant moment several times until they fail. A special jig has been designed for this purpose by the author, who also analyzes the strains in the plastically curved part of the bar. After conducting a number of tests with several materials, he finds there is a linear relationship between the angle of bending and the inverse of the number of bending moment applications for failure, and, also, a linear relationship between this last quantity and the imposed maximum elongations of the extreme fibers. The product of the number of load repetitions before failure and the imposed maximum elongation at the extreme fiber is a constant and a characteristic of the material. Author claims results obtained using this method of testing present much smaller dispersion of values than those obtained in the commonly used bend tests.

Author applies his method to studies in crystals and in short specimens with a small number of crystals, and finds that the dispersion of the results is proportional to the degree of inhomogeneity of the granular structure of the metal.

A. J. Durelli, USA

Failure, Mechanics of Solid State

(See also Revs. 2007, 2016, 2026)

2043. **Tsobkallo, S. O., Plasticity phenomenon at brittle failure of soft steel** (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 6, 844–847, June 1951.

At temperatures ranging from −180 to 0 C, bending impact tests are carried out on notched cylinders of low-carbon steel with 0.2% C. Temperature of transition from ductile to brittle fracture is −40 C. Fracture surfaces are examined by back-reflection x-ray method. On photographs obtained with $K\alpha$-radiation of cobalt anode, intensity and width at half maximum intensity of $K\alpha$-line are measured. The latter decreases with decreasing temperature of fracture. In comparison with undeformed material, a considerable broadening of interference line is observed on specimen broken at temperature of liquid oxygen. From this, author infers that plastic deformation always appears at brittle fracture, for which he estimates the value for the temperature of liquid oxygen to be about 5%.

Heinrich Mussmann, Germany

Material Test Techniques

(See also Revs. 1969, 2027, 2034, 2042, 2047, 2059, 2187)

2044. **Kennedy, A. J., A constant stress device adjustable for specimen length,** *J. sci. Instrum.* **29,** 2, 40–44, Feb. 1952.

Rational design of constant-stress device similar to that of Andrade and Chalmers. Present design incorporates secondary correction profile and loading weight that considerably reduces error and extends use to a wide variation in specimen lengths and deformations. Error in stress for specimen of length up to 50% variation from design length is within 0.4%. This error is reduced to about 0.02% for variations in length of about 1%.

George H. Lee, USA

2045. **Apblett, W. R., and Pellini, W. S., A recording dilatometer for high temperatures,** *Trans. Amer. Soc. Metals* **44,** 1200–1214, 1952.

A novel method is described which permits extension of dilatometry to temperatures considerably in excess of the present practical limits of 1800 F (980 C) with concomitant flexibility of heating rate. The method consists of electrical resistance heating of a small rodlike specimen. Temperatures and resulting dilations are recorded by means of thermocouples and SR-4 clip gages, respectively; these are affixed to the specimen and suitably connected to high-speed response-recording instruments. Heating rates of 500 F/sec are readily attainable by this method.

Data are presented of the linear expansion of various high-temperature alloys to 2400 F (1315 C), molybdenum to 3000 F (1650 C), and tungsten to 4760 F (2630 C). The applicability of the dilatometer to transformation studies over a wide range of temperatures is likewise demonstrated.

From authors' summary

2046. **Tenot, A., Note concerning internal damping of materials and its determination by means of the Amsler electromagnetic pulsator** (in French), *Mém. Soc. Ing. civ. Fr.* **104,** 5/6, 184–191, 1951.

Author describes briefly a damping capacity measuring machine known as Vibrophore, and derives mathematical relationships for calculating the damping from the vibrations-decay record. Corrections are made for damping due to the machine. Sample damping data are presented for metals and plastics as a function of alternating tension stress.

Alexander Yorgiadis, USA

Mechanical Properties of Specific Materials

(See also Revs. 2007, 2029, 2030, 2031, 2032, 2033, 2034, 2037, 2065, 2163, 2235)

2047. **Krisch, A., Notch-tension tests on hardened structural steel** (in German), *Arch. Eisenhüttenw.* **22,** 11/12, 395–400, Nov./Dec. 1951.

Results of tensile tests and of notched-bar impact tests under carefully controlled conditions are reported for specimens taken from two carbon steels and from eight alloy steels. The tensile tests were made on specimens of circular section without a notch, with a semicircular notch, and with a 60° V-notch. The tests were made at 20 and at −70 C. The specimens with the semicircular notch showed tensile strengths at failure which were about 1.38 times that for the unnotched specimens. In the case of specimens with the V-notch, the ratio of the two tensile strengths increased with increasing notched-bar impact strength from about 1.4 to about 1.9. Lowering of the temperature from 20 to −70 C produced the same increase in tensile strength (5 to 15%) in the

specimens with the semicircular notch as in the unnotched specimens. In the case of the specimens with the V-notch, the increase was smaller and more erratic (sometimes negative), showing that a low-temperature tensile test of these specimens is relatively severe. The author concludes from the tests that the "notch sensitivity" (Trennempfindlichkeit) as defined by Kuntze varies directly (presumably inversely) with the notched-bar impact strength (Kerbschlagzaehigkeit).

Walter Ramberg, USA

2048. **Johnson, A. E., Creep under complex stress systems at high temperatures,** *Aircr. Engng.* **24,** 275, 6–16, Jan. 1952.

Previously, author reported tests on different representative metallic alloys of basic groups used in practice, and in this article he concludes with results on a nickel-chromium alloy (Nimonic 75) at 550 and 650 C. Work fills the great need for data in this field and enables a critical examination of the theoretical work done by others.

Author concludes that the creep behavior of all materials can be represented by an equation of the Saint Venant-von Mises type generalized as previously reported [AMR **3,** Rev. 2669], but two varieties are indicated. The first variety is associated with stresses of low and moderate value and with closely isotropic behavior of the material. In this case, the equation involves only a simple power function of the stresses. The second variety is associated with pronounced anisotropy and occurs at high stresses in relation to the temperature. The nature of anisotropy depends on the actual stress system. The relations between creep rate and stress involves, in this case, the use of complex power functions of stress.

Appendix 1 concerns the results from the tests with Nimonic 75, and appendix 2 treats the effect of hydrostatic stress upon the complex stress-creep properties, whereby the tests do not seem to give any modification of creep values. Appendix 3 gives some examples of application of creep rate-stress relations obtained in the experimental work.

Reviewer suggests that a principal, important work is now partly fulfilled, but before the work is ended it seems desirable to complete it with some tests of longer duration than 150 hr on the different materials.

Sven Edling, Sweden

2049. **Grover, H. J., Hyler, W. S., and Jackson, L. R., Fatigue strengths of aircraft materials,** *NACA TN* 2639, 22 pp., Feb. 1952.

This report presents results of axial-load fatigue tests on notched specimens of three sheet materials: 24S-T3 and 75S-T6 aluminum alloys, and normalized SAE 4130 steel. Each specimen was notched by edge notches designed to have a theoretical stress-concentration factor of 1.5. Tests were run at four levels of nominal mean stress: 0, 10,000, 20,000, and 30,000 psi.

Results of these tests extend information previously reported from tests on unnotched specimens and tests on specimens more severely notched, and afford data on the variation of fatigue-strength reduction with notch severity.

From authors' summary

2050. **Frey, D. N., and Freeman, J. W., Fundamental effects of cold working on the creep resistance of an austenitic alloy,** *J. Metals* **3,** 9, 755–760, Sept. 1951.

Effects of cold-working on low-carbon N-155 alloy investigated by x-ray diffraction studies correlated with creep rates under 50,000 psi at 1200 F. Steady-state creep rates were lowered with reductions in cross section up to 25% at 80 F. For 15% reduction in section, creep rates are low for rolling temperatures up to 1500 F, when large increases in creep rate occur. Samples annealed following different degrees of rolling reduction show increasing creep rate with annealing time. The widths of the (220) diffraction line of austenite were observed for all samples, and this width was taken to be a measure of residual elastic strains. Correlation of line widths and creep rates as observed is considered as evidence that the effects on creep rate are the result of such residual strains.

Henry A. Lepper, Jr., USA

2051. **Sinclair, G. M., and Dolan, T. J., Some effects of austenitic grain size and metallurgical structure on the mechanical properties of steel,** *Proc. ASTM* **50,** 587–616, 1950.

Further important information on the relationship between microstructure and mechanical properties of steel is provided by this paper. From the experimental results, authors conclude that austenitic grain size has little effect on the properties of the tempered martensitic steels, whereas large effects were found in the steels heat-treated to produce pearlitic microstructures. In the latter steels, an increase in austenitic grain size was found to result in (a) an increase in the yield ratio in tension, (b) a decrease in the fatigue notch sensitivity of the steel, and (c) a decrease in energy absorption in notched-bar impact tests over the entire range of temperature investigated. Authors point out that "notch sensitivity" in fatigue and "notch sensitivity" in impact are affected in entirely different ways by changing austenitic grain size, indicating that these characteristics are not directly related and should not be used interchangeably. Results reported by authors confirm the generally recognized fact that, in general, the tempered martensitic type of microstructure results in mechanical properties superior to those obtained with pearlitic microstructures.

In considering the detailed experimental results, the reader should note that "pure" microstructures were not obtained in every instance. For example, the 1045 steel does not possess sufficient hardenability to permit a completely martensitic microstructure on quenching bars of the size employed. Furthermore, there are possibly significant differences among the "pearlitic" microstructures produced in the different steels. To some extent, therefore, the generality of the conclusions must be qualified on the basis of the specific microstructures obtained.

W. T. Lankford, Jr., USA

2052. **Osipov, K. A., Investigation of plastic and other properties of alloys of the system iron-nickel-tantalum** (in Russian), *Izv. Akad. Nauk. SSSR Otd. tekh. Nauk no.* 6, 848–851, June 1951.

Specific electric resistance, hardness, and deflections of rods of several iron-nickel-tantalum alloys (with 5, 10, 15, 20, 25, 30, and 35% tantalum) were determined. All specimens were cast, annealed for 48 hours and quenched in oil, and rods were subjected to bending in a centrifugal testing machine (method of I. I. Kornilov). Curves of equal specific electric resistance and of equal deflections in bending of alloys in the domain of the λ solution are given. In the domain of the γ solution, the deflections in bending diminish, if content of Fe is increased or if the limit of specific solubility is approached. Transition in the two-phase domain of $\gamma + Fe_2Ta$ is accompanied by smaller deflections, while transition in the domain of $\gamma + Ni_3Ta$ gives greater deflections.

Anton Kuhelj, Yugoslavia

2053. **Meyers, S. L., Thermal expansion characteristics of hardened cement paste and of concrete,** Nat. Res. Counc. Highway Res. Board, Proc. 30th Ann. Meeting, 193–203, 1951.

Factors influencing the thermal expansion of hardened, neat Portland cement and of concrete were investigated, primarily as a contribution to problems in concrete durability. Increases in relative humidity up to about 70% caused the thermal-expansion

coefficient of the cement to increase. Further increases in humidity lowered the coefficient to a minimum at 100% relative humidity. This characteristic was unchanged by increasing aging, except that the maximum coefficient was somewhat lower and occurred at a relative humidity somewhat below 70%. The humidity dependence of the coefficient was primarily a function of the extent of cement hydration, with possibly some influence from other characteristics of the internal structure. There was little humidity dependence in highly hydrated specimens. Aggregates in the cement tended to restrain changes in its expansion, making variations in thermal expansion with humidity much smaller in concrete than in cement.

W. H. Duckworth, USA

2054. Wright, R. H., and Hayward, A. M., Kinetics of the thermal decomposition of wood, *Canad. J. Technol.* **29**, 12, 503–510, Dec. 1951.

As part of a study of methods for producing fuel gas from waste wood, authors studied the effect of particle size and of temperature on the rate of decomposition of western red cedar and western hemlock. A variation from usual commercial processes was used by suddenly introducing small pieces of wood into an atmosphere of nitrogen at high temperature. Wood was cut into cubes—ranging from 19-mm to 3-mm edge—which were dropped into heated atmosphere in oven-dry condition. Rate of decomposition was followed by means of a calibrated recording diaphragm manometer.

Authors found reaction rate to be of the one-half order for cubical specimens, with the rate constant being directly proportional to the specific surface and to the temperature. The decomposition of disks cut across the grain is more nearly a zero-order process than that of disks cut parallel to the grain. This might be expected from the anisotropic nature of wood. Proportionality constants are the same for both species tested. Good agreement between calculated curves and experimental data was obtained.

Benjamin S. Bryant, USA

2055. Nistica, F., Sprague, B. S., and Work, R. W., A high-speed recording yarn tensiometer, *Text. Res. J.* **22**, 2, 99–110, Feb. 1952.

A yarn tensiometer is described which is capable of detecting and recording variations in tension occurring at a rate of the order of 100 cps. Tension variations of a considerable magnitude at such frequencies have been shown to exist in yarn running on standard textile machinery. From authors' summary

2056. Stauffer, W., Some problems of heat-resistant steels from the consumer's viewpoint (in German), *Schweiz. Arch.* **17**, 12, 353–364, Dec. 1951.

The scattering of the creep-stress values according to DVM standards is illustrated by means of test results on different samples of the same material under constant conditions. Whereas average creep velocities vary considerably, creep stresses show relatively small scatter. Results of long-time creep tests on molybdenum cast steel show properties with varying molybdenum content at high temperatures.

Test results on tube samples submitted to internal pressure are compared with similar tensile creep tests, as a comparison between different stress states. Creep-test results on different cast steels and on welds are given. Influence of combustion gases with vanadium content on heat-resisting steels is treated, and means of minimizing corrosive effects of vanadium components are indicated. Methods of checking heat-resisting castings are shown.

E. Haenni, Switzerland

2057. Meyer, M. A., and Blok van Laer, K. J., Plastic deformation and the Meyer constants of metals, *Nature* **169**, 4293, 237–238, Feb. 1952.

Authors take issue with the conclusion of Finniston, Jones, and Madsen [*Nature* **164**, p. 1128, 1949] that noncubic metals (including Mg, U, Sn, Zn, Ga) differ from cubic metals (including Fe, Cu, Al) in not obeying the relation between Meyer hardness number for the metal in a cold-worked state and the stress-strain curve for large strains for the metal in the same state which was derived by Tabor [AMR **1**, Rev. 654]. They present a large number of measurements of Meyer hardness number for the noncubic metals Zn, Cd, Sn which are in good accord with this relation up to strains of the order of 0.20. For these measurements the metal was annealed to begin with, was cold-worked a prescribed amount by rolling it down, and the impressions were made immediately afterward to minimize the effects of recovery. Authors suggest that the effects observed on noncubic metals by Finniston, Jones, and Madsen may be due to recovery rather than being characteristic of noncubic metals.

Walter Ramberg, USA

2058. Kuntze, W., Characteristics of the brittleness tendency of steel in notch tension and notch tension impact tests (in German), *Arch. Eisenhüttenw.* **22**, 11/12, 387–392, Nov./Dec. 1951.

Author has conducted an extensive experimental program with different alloyed steels, unalloyed steels, and nonferrous metals in various forms and states. Reduction-of-area values at fracture are obtained from an ordinary tension, a notch tension, and notch-tension impact test for a particular material in a particular form and state. From comparisons of these reduction-of-area values the concepts of natural, latent, and form brittleness are derived as the three basic factors that cause the brittle tendency in a material. The first of these is the well-known brittleness exhibited in the ordinary tension test by a small reduction-of-area value at fracture. A polished specimen reflects a characteristic structure associated with natural brittleness.

Latent brittleness, as defined by author, is produced by very small deposits of foreign material in the lattice structure of the base, which are mechanically active just in the beginning of the test. The deposits are so small that no phase change can be detected microscopically. The ordinary material-testing procedures therefore conceal latent brittleness. Author detects it in the ratio of the reduction of area at fracture in the notch-tension impact bar to that in the notch tension bar, the amount of latent brittleness varying inversely with this ratio. It is pointed out that this basic factor brings together such well-known ones as impact brittleness, aging brittleness, cold brittleness, etc. Interesting is the fact that ductile low-carbon steel displays the strongest latent brittleness.

Comparisons of the reduction of area at fracture in the notch tension bar with that in the ordinary tension bar reflect the form brittleness factor. It is pointed out that this type of brittleness includes not only the effects of body shape, but also material peculiarities.

Cases from practice, exhibiting damage due to brittle failure, have also been investigated by author according to the above procedure and breakdown.

Julius Miklowitz, USA

2059. Schmidt, H., Some measurements on barium titanate (in German), *Akust. Beihefte* no. 2, AB83–AB88, 1952.

For circular piezoelectric disks, formulas are given from which it is possible to calculate the mechanical constants of the disk material from electrical measurements. The dependence of temperature upon dielectric constant, the electrical loss factor,

elastic modulus, mechanical loss angle, and the piezo-modulus of $BaTiO_3$ and the mixed ceramic $BaTiO_3$-$PbTiO_3$ have been measured, and are represented diagrammatically. The numerical value of the mechanical and electrical constants of barium titanate are collected in a table. From author's summary

2060. **Staples, R. T., Flash annealing of light alloys,** *J. Inst. Metals* **80,** 323–334, 1951–1952.

Results and discussion of merits of single-sheet annealing in conveyor-type furnace. Rapid heating results in fine grain-size on recrystallization. George V. Smith, USA

2061. **Sinnott, M. J., and Shyne, J. C., An investigation of the quenching characteristics of a salt bath,** *Trans. Amer. Soc. Metals* **44,** 758–768, 1952.

The quenching constant H, used to determine the severity of a given quench, has been determined for quenching from a high-temperature salt bath at 1550 F (845 C) into low-temperature unagitated salt baths between 385 and 725 F (195 and 385 C). The values of H have been found to be comparable to those reported for oil quenching with good to violent agitation.

Variations in the values of H, depending on where temperature measurements are taken, are shown to be due to the existence of a surface-film resistance. The instantaneous value of this surface-film heat-transfer coefficient has been shown to be a function of the temperature difference between the metal surface and the temperature of the quenching bath ($T_S - T_B$). For the test conditions investigated, the value of this instantaneous coefficient in Btu/sq ft/hr/°F is given by the following equation: $h_f = 93e^{0.0015(T_S - T_B)}$. From authors' summary

2062. **Asada, H., Tanaka, E., and Koike, K., Elastic anisotropy of $\alpha + \beta$ brass strip** (in Japanese), *Rep. Inst. Sci. Technol. Tokyo* **6,** 1, 19–25, 1952.

The elastic anisotropy of $\alpha + \beta$, α and β brass, cold-rolled and annealed, are measured. Results are:

(1) Elastic anisotropy of cold-rolled $\alpha + \beta$ brass does not depend on the β phase but the α phase. When the α phase is cold-rolled the preferred orientation is formed gradually, but the β phase cannot form the preferred orientation and breaks down at the cold-rolling degree of 30% or more.

(2) When cold-rolled $\alpha + \beta$ brass is annealed at 200 C, Young's modulus increases in each direction and this is caused by its phase.

(3) Elastic anisotropy of recrystallized $\alpha + \beta$ brass strip (annealed at 400 C) decreases, and the value of Young's modulus in each direction becomes the mean value of Young's modulus of α and β brass strip.

(4) The curve of the Young's modulus calculated from the pole figure of β brass strip is very similar to the observed curve, but about α brass both curves are different because the pole figure of β brass shows clear preferred orientation, while the pole figure of α brass is not as clear as the former.

From authors' summary

2063. **Asada, H., and Tanaka, E., Study on the softening of cold-rolled β-brass strip by low temperature annealing and on its rolling texture** (in Japanese), *Rep. Inst. Sci. Technol. Tokyo* **6,** 1, 13–17, 1952.

In order to study the properties of cold-rolled $\alpha + \beta$ brass strip, the properties of cold-rolled β brass strip were studied.

Cold-rolled β brass strip softened even at room temperature and almost entirely softened by annealing below the recrystallization temperature.

Though the critical reduction of cold rolling for β brass was 20%, author applied the above-mentioned properties of softening to the rolling process and got the plate of which reduction was 80% and then examined its rolling textures from etch pits. It had (100) [110] as the main preferred orientation, and (111) [112], (111) [110], (111) [134], and (112) [110] as the second.

These textures were similar to the rolling texture of other body-centered cubic metals, e.g., Fe, W, and Mo. When the cold-rolled plate recrystallized, these textures became at random. From authors' summary

Mechanics of Forming and Cutting

2064. **Chao, B. T., and Bisacre, G. H., The effect of speed and feed on the mechanics of metal cutting,** *Instn. mech. Engrs. Proc.* **165** (*W.E.P.* 63), 1–9, 1951.

Speed and feed have some effect on the inertia forces within the chip, but since these forces are small compared to the force of deformation, they can be neglected. Speed and feed do have, however, considerable influence on the physical behavior of the material being cut, due to their effect on the strain rate and temperature.

With increased strain rate, the stress-strain curve approaches that of an ideally plastic material which does not work-harden. Metal in the shear zone retains the properties it had at the general temperature of the workpiece, whereas metal in the friction zone has its properties modified by temperature.

To confirm these beliefs, a series of experiments were conducted on mild steel and copper, over a wide range of speed, feed, and tool angles. In presenting their data, authors often find it convenient to plot their results on a graph in which the thermal number is used as an abscissa. Thermal number is proportional to the product of the cutting speed and chip thickness before removal from the workpiece. The latter is measured as feed per revolution of lathe spindle. Authors also plotted data which show the influence of the thermal number on the extent of the built-up nose. Bernard W. Shaffer, USA

2065. **Anonymous, Machining of stainless steels,** New York, Metal Cutting Tool Inst., 27 pp., 1951.

This report is a comprehensive résumé of the basic requirements for machining all types of stainless steel. It includes a section on composition and general characteristics of wrought stainless steels; practical considerations in machining stainless steel; tool designs and materials for milling, drilling, reaming, and threading stainless steels in addition to a section considering coolants and lubricants for the various operations. Comparative values of power requirements, cutting speeds and feeds are also included.

The article is highly recommended for furnishing the fundamentals for machining of stainless steels. J. F. Snider, USA

2066. **Armitage, J. B., and Schmidt, A. O., Force and performance measurements in metal cutting** (in German), *Werkstattstech. Maschinenb.* **42,** 1, 1–7, Jan. 1952 = *Tool Engineer* **27,** 4, 5; 36–40, 50–54; Oct., Nov. 1951.

A summary and analysis of power and force measurements at various cutting speeds from the study of 24 references are presented. Authors deduce that, for practical purposes, tool forces remain constant and are not diminished by higher cutting speeds; that there is a linear increase with power requirement at the tool with an increase in speed; that tool wear increases sharply above a limiting speed; that, generally, running the machine tool at high speed with the expectation of removing a large amount of metal with a decrease in power is a false hope, although an in-

creased rate of production and better surface finish may be definitely achieved by higher cutting speeds.

Reviewer notices a significant omission of analysis of data obtained by Merchant, Zlatin, Field, Fersing, Shaw, Marshall, Cook, Zylstra, and others who have found substantial decreases in tool forces and power with higher cutting speeds, under definite cutting conditions.

Dimitri Kececioglu, USA

2067. Lenz, D., Investigation of a new warm deep drawing method (in German), *Arch. Eisenhüttenw.* **22**, 7/8, 215–224, July/Aug. 1951.

An elevated-temperature deep-drawing technique is described that permits drawing larger blanks and forming larger parts from aluminum alloys than is possible by conventional room-temperature techniques. The following zone heating was found to be particularly effective: Heated drawing ring and blank hold-down plates; water-cooled drawing ring and room temperature; or water-cooled punch. By this technique, reduction ratios (blank radius/part radius) as large as 3.9 were found possible, as compared to a maximum ratio of approximately 2 at room temperature. Blankholder test temperatures up to 400 C were tried. Maximum drawing limits were observed at from 300 to 400 C. Lubrication and blankholder pressure were found to be critical.

An analytical discussion of the effects of elevated temperature is presented.

William Schroeder, USA

2068. Stabler, G. V., The fundamental geometry of cutting tools, *Instn. mech. Engrs. Proc.* (*W.E.P.* 63), 14–21, 1951.

The geometry of oblique cutting with a tool having a plane cutting face is considered. Simple shear over a shear plane is assumed, so that the motion is determined by the tool geometry, the directions of motion of the work and the chip relative to the tool, and the shear plane angle. Various definitions of rake angles are discussed, and a simple relation between the directions of motion of the work and chip relative to the tool is deduced from experimental results. This is used to study the chip shape and the geometry of strain in the chip. Machining force components are also considered, and the combination of these with the chip-motion law mentioned above determines a relation for the shear plane angle. In the limit for normal machining, it gives a result different from that due to Merchant, which is based on minimum energy considerations. Reviewer believes that the ellipsoid of stress determined should be called the ellipsoid of strain, since for deformation under nonhomogeneous stress or with large rotations the two do not coincide.

E. H. Lee, USA

2069. Bendixen, I., Vibration in milling machines (in Danish), *IngenVidensk. Skr.* no. 2, 48 pp., 1951.

The main object of these investigations was to find out the influence in respect to vibration of the knee, saddle, and overarm of a milling machine which are clamped. The tests were run with three different plain milling cutters, which machined a workpiece of the dimensions 64 × 64 × 750 mm under normal cutting conditions.

The amplitude of the vibrations grew evenly with increasing distance between milling cutter and pick-up, and the smallest amplitudes occurred when overarm as well as saddle and knee were clamped. Loosening of the knee means an increase in the amplitude of between 60 and 65%. It was strange to notice, however, that in a number of tests a 4 or 5% smaller amplitude was found when the knee was released from the column of the machine than when it was clamped, and the maximum amplitudes were ascertained when milling took place near the center of the workpiece.

On the basis of W. Klein's Fourier analysis of the milling process, an expression for the amplitude of force (P_{K_0}) is developed—the amplitude of the κ_{th} higher harmonic with cylindrical cutter.

It is demonstrated, theoretically on the basis of this equation, and empirically, that the amplitude of the vibrations increases with the feed. By changing the width of cut when milling with a cylindrical cutter with spiral grooves, it was demonstrated that a plain milling cutter should be designed so that the equation $\kappa \cdot \mathrm{tg}\lambda \cdot z \cdot b/D = \pi, 2\pi, 3\pi \ldots$ is satisfied (b width of cut (mm), and λ helix angle, i. e., the angle between the cutting edge and the axis of the milling cutter).

For the dependency of the amplitude on the cutting speed, the curves show a relative minimum between 15m/min and 45m/min and a declining course from 100m/min to 350m/min. A series of comparative tests with down-milling and up-milling showed that down-milling gives smaller amplitudes than up-milling even if the machine, which, however, must be in good condition, is not particularly arranged for down-milling

From the author's English summary

2070. Anonymous, Efficient milling, New York, Metal Cutting Tool Inst., 28 pp., 1950.

Hydraulics; Cavitation; Transport

(See also Rev. 2091)

2071. Ippen, A. T., Knapp, R. T., Rouse, H., Bhoota, B. V., and Hsu, E.-Y., High-velocity flow in open channels, a symposium, *Trans. Amer. Soc. civ. Engrs.* **116**, 265–363, 1951.

This symposium summarizes the development in a relatively new field of practical interest to many engineers. Since the field is new, virtually complete development of the basic theory is given, which makes the publication useful as a general reference on the subject.

In the first paper, Ippen outlines clearly the basic principles of high-velocity flow, i.e., flow with a velocity exceeding the wave velocity, giving a straightforward derivation of the theory and a physical interpretation of the results. Examples of the application of the theory to some important elementary cases are discussed in detail to illustrate the principles. Calculation of flow and disturbance patterns in practical cases by using the Busemann shock-polar diagram and method of characteristics is explained in detail.

In the second paper, Knapp analyzes flow in channel curves for supercritical flow and presents several ways of designing such curves for rectangular channels, based on the principles outlined in the first paper. The two general methods considered are banking and wave interference, and design criteria are given for each method. In the interference method, diagonal sills, compound curves, and spiral transitions are used to produce the proper wave patterns, and their relative merits are discussed. Extensive experimental data confirm the design procedures and illustrate the impracticability of using any but rectangular channels in high-velocity curves.

The third symposium paper by Ippen and Dawson endeavors to show, on the basis of experimental evidence, that the basic principles of supercritical flow can be applied in a satisfactory manner to the design of typical channel contractions, and that solutions may be found with a minimum amount of surface-disturbance patterns. Furthermore, the magnitude of standing waves may be predicted adequately, as well as their location within the channel contraction. The method of eliminating waves in the downstream channel is discussed for a basic form of contraction.

In the last paper Rouse, Bhoota, and Hsu give an introductory discussion of supercritical flow in diverging channels and their

discuss the matter of channel design under these three headings: (1) Surface configuration at abrupt expansions; (2) efficient curvature of expanding boundaries; and (3) elimination of disturbances at the end of transitions. The extent of the agreement between elementary wave theory and experimental measurement is shown, and the results are presented in the form of generalized diagrams convenient for rapid exploration and preliminary design.

Vito A. Vanoni, USA

2072. **Green, J. E., The pressure surge in oil pipelines,** 3rd World Petr. Congr., The Hague 1951, Proc. Sect. 9, 7–21.

Paper deals with pressure surges in oil pipe lines, whose quantitative characteristics are quite different from conventional water-hammer phenomena because (a) flow stoppage time is short with respect to pipe reflection time, and (b) friction losses are not small with respect to pressure surges.

After describing a step-by-step solution, two mathematical solutions are given: first, one by Ludwig and Johnson corresponding to rough schematization; the second one for laminar flow, strictly derived from corresponding case of electric transmission lines. Electric analogs of phenomenon given and discussed by author are crude with respect to long-known analogy with transmission lines (as pointed out by Lefebre in discussion).

Giuseppe Evangelisti, Italy

2073. **Randall, L. N., Rocket applications of the cavitating** Venturi, *J. Amer. Rocket Soc.* **22**, 1, 28–31, Jan.-Feb. 1952.

Use is made of the fact that, after cavitation occurs in a Venturi meter, a further reduction in the downstream pressure does not increase the rate of flow. Cavitating Venturis have been successfully used for accurate flow control, thus eliminating costly flow-control equipment. Design considerations are presented. Author suggests other uses for cavitating Venturis such as temperature control.

W. M. Owen, USA

2074. **Evangelisti, G., On the stability of large oscillations in surge tanks** (in Italian), *Energia elett.* **28**, 12, 673–687, Dec. 1951.

The first part of the paper is a recapitulation of the theory of characteristic lines and singular points of the differential equation governing the systems with one degree of freedom. A topologic research in the plane of the phases leads rapidly to the individuation of stability conditions for small oscillations.

The approximate integration of the nonlinearized equation (for large oscillations) then makes evident the existence of an unstable limiting cycle, and therefore the possibility of the generation of large oscillations with character of instability. The conditions under which such oscillations cannot take place are established in a very simple approximate formula. Results of the study are compared with those obtained by other authors (Schüller, Karas, Jaeger, Frank).

Duilio Citrini, Italy

2075. **Doolittle, A. K., Studies in Newtonian flow. II. The dependence of the viscosity of liquids on free space,** *J. appl. Phys.* **22**, 12, 1471–1475, Dec. 1951.

In a prior paper [see AMR 5, Rev. 1114], author expressed the conjecture that not the temperature but another physical property depending on temperature influences the viscosity primarily. He concluded this from the complexity of the viscosity-temperature relations, which reproduce the values of viscosity within the limits of accuracy of measurements. In the present paper, he shows that it is possible to interpret the viscosities of liquid normal paraffins as a simple function of relative free-space defined as $v_f/v_0 = (v - v_0)/v_0$; v_f is spec. vol. of free-space, v spec. vol. of the liquid, v_0 spec. vol. extrapolated to absolute zero without change of phase.

Methods for evaluating v_0 hitherto proposed in literature are impractical. Either they demand knowledge of critical properties or they prove to be incorrect or uncertain. Author states that the values of specific volumina of the homologous series examined by him obey the following simple formula

$$\ln v = a/m + b \text{ with } \ln a = C_1T + C_2,\ b = C_3T^{C_4} + C_5$$

where m is molecular weight. Therefore he is able to determine v_0 and by it the relative free-space. From plotting the logarithm of viscosity vs. reciprocal of relative free-space straight lines, there results

$$\ln \eta = B(v_0/v_f) + A \text{ or } \eta = A \exp\,[B/(v_f/v_0)]$$

Thus author succeeds in finding a relation which reproduces the measured values of viscosity satisfactorily over a range of temperatures extending from boiling point to less than 100 C above freezing point of liquid. Because changes in relative free-space result from changes in pressure as well as from changes in temperature, dependence of viscosity on pressure ought to obey the same formula. Unfortunately, no data are available hitherto to prove that.

Ulrich Rost, Germany

2076. **Doolittle, A. K., Studies in Newtonian flow. III. The dependence of the viscosity of liquids on molecular weight and free space (in homologous series),** *J. appl. Phys.* **23**, 2, 236–239, Feb. 1952.

In a previous paper [see AMR 5, Rev. 1114, and preceding review] author found that dependence of viscosity of n-alkanes, except values in the neighborhood of the freezing points, upon relative free space can be represented by

$$\ln \eta = B(v_0/v_f) + \ln A;\quad v_f/v_0 = \text{relative free space}$$

In the present paper, author tries to give a physical interpretation of this equation, which in reviewer's opinion is incorrect. First, viscosity is either the property of a material to produce in itself a resistance when the material is continuously deformed under stress, or a material constant relating the stress with the rate of strain, but by no means the resistance itself. Moreover, the resistance does not arise from moving of the molecules past each other against the cohesive forces, because the molecules withdraw from molecules in the neighborhood in the same proportion as they approach other molecules. Rather, the resistance originates from an impulse transfer.

Plotting the viscosity straight lines into the $\ln \eta$ vs. v_0/v_f diagram, author states that the lines belonging to alkanes with a molecular weight between 100 and 240 intersect the vertical axis at nearly the same point -7.6500. Employing this average intercept necessitates corresponding adjustments in the slopes of the lines. Therefore, assuming $\ln A$ to have the value -7.6500, the calculated values are no longer as precise as the accuracy of measurements. Calculating the slopes B of the lines, author establishes that their dependence upon molecular weight can be represented by $B = C \cdot \exp(-Km^{1/20})$. This yields the following function which gives the dependence of viscosity upon molecular weight and relative free space

$$\ln \eta = C \cdot \exp(-K/m^{1/20})v_0/v_f + \ln A, \text{ or for } n\text{-alkanes}$$

$$\ln \eta = 2401 \cdot \exp(-10.239/m^{1/20})v_0/v_f - 7.6500$$

Though this relation gives values which do not differ from the measured by more than 1%, it is rather impractical—even author emphasizes that—because it is only valid for n-alkanes with a molecular weight between 100 and 240; further, it does not represent the values within the accuracy of measurements.

Ulrich Rost, Germany

2077. Sweeney, D. C., Preliminary investigation of hydraulic lock, *Engineering* **172;** 4474, 4476; 513–516, 580–582; Oct., Nov. 1951.

Author describes an investigation into the characteristics of the phenomenon known as hydraulic lock occurring in valves and pumps of high-pressure hydraulic systems which is manifested, when the piston is stationary, by a very considerable increase in friction over a period of time. The influence of hydraulic pressure on the locking force, the build-up of the force with time, and the radial movement of a piston in a cylinder were studied. The results indicate the importance of macroscopic surface irregularities of the components in the form of slight deviations from parallelism, and an explanation is put forward based on the tapering fluid films in the clearances to which these irregularities give rise. Circumferential grooving of the piston lands substantially reduced the locking force.

Although full confirmation of the suggested theory was not possible, it is felt that the information obtained helps toward a better understanding of the underlying causes of the phenomenon.

André Leclerc, Canada

2078. Ball, J. W., Model tests using low-velocity air, *Proc. Amer. Soc. civ. Engrs.* **77,** Separ. no. 76, 13 pp., June 1951.

After a short historical introduction, author discusses advantages (low costs) and disadvantages of replacing water with low-velocity air in hydraulic model tests. Attention is paid to limitation of incompressible flow formulas, generally at $^1/_4$ sonic speed. This limit, however, is influenced by local conditions in tests, e.g., accuracy of test instruments. Paper gives general view of problems studied in this way and experiences of author in using method for determining characteristics of hydraulic outlet structures, e.g., errors by improper turbulence in flow.

H. Thygesen Kristensen, Sweden

2079. Supino, G., Air-water analogy and the study of hydraulic models (in Italian), *Energia elett.* **28,** 11, 613–618, Nov. 1951.

Highly accurate aerodynamic measuring instruments, particularly for turbulence, together with the practical advantages of large models, low structural and power requirements, as well as absence of wetting problem or need for absolute tightness in air system are advantages realized to accrue from testing water equipment and flows by means of air analogies, where the atmosphere serves as both a convenient supply reservoir and catch basin. Problems of compressibility of air are avoided by using low-velocity airstreams (see preceding review).

Author presents expository article serving to focus attention on specific applicability of dimensional analysis to problems met in this rebirth of interest concerning such air-water tests. Although no novel basic findings are unearthed, review is thorough, concise, and particularly valuable in showing limitations of the several modes of theoretical approach of which the Buckingham pi-theorem is most general but least practical. It is pointed out that there are more cases in which the water-air analogy is possible than is commonly supposed. Not only is it useful for study of diffusion of air jets and the application of such results to submerged sluice gates, but even more complex free-surface phenomena (where density is discontinuous across the free surface) can be handled, under proper restrictions. Other useful applications easily spring to mind: Penstock design and operation information, losses and characteristics of water-distribution systems, hydraulic pumps, turbines, fluid meters, intricate passages in valves impellers, etc. (In many cases, gravity, i.e., Froude number, is neglected; though usually the viscous effects are important, but even then, Reynolds number equivalence is not always necessary and, when it is, a *reduction* in model size to $^1/_{10}$ of original is often feasible with moderate air velocities.)

R. H. Cramer, USA

Incompressible Flow: Laminar; Viscous

(See also Revs. 2072, 2124, 2215)

2080. Loĭtsianskiĭ, L. G., Fluid and gas mechanics [**Mekhanica zhidkosti i gaza**], Moscow-Leningrad, Gos. Izd. Tekh.-Teor. Ltd., 1950, 676 pp.

Book is based on a lecture course given at the Kalinin Institute of Technology in Leningrad. It aims to give a systematic presentation of the fundamentals of fluid mechanics, leaving applications almost completely aside.

After a historical introduction whose main features are conventional for the period up to 1900, but which hardly mentions any non-Russian work thereafter, the following topics are discussed: Chapter 1, The notion of fields and the kinematics of continuous media; chapter 2, Equations of equilibrium of continuous media; chapter 3, Dynamics of ideal liquids and gases—General theorems. These three chapters (one fifth of the book) give a very solid, general account of the classical fundamental equations of fluid mechanics. Free use is made of vector notation and vector analysis.

Chapter 4, One-dimensional flow of an ideal fluid. A detailed account of one-dimensional isentropic flow of a perfect gas, and of shock waves. Chapter 5, Steady motion of a fluid—Plane motion of an incompressible fluid. After a brief discussion of three-dimensional problems, author gives extended presentation of plane irrotational-flow problems, including airfoil and cascade theory and some free streamline problems in terms of the theory of functions of a complex variable. Chapter 6, Plane steady motion of a compressible gas. Beginning with the linearized subsonic and supersonic flow past a wavy wall, author then discusses the hodograph method, briefly describes some transonic flow patterns, discusses the characteristic network of nonlinear supersonic flow in the physical plane and the hodograph and the oblique shock theory. Chapter 7, Steady motion in space. Source distributions, vorticity distributions, axially symmetric motion of incompressible and subsonic fluids, generalized motion of bodies and the concept of apparent mass tensor, lifting-line theory are covered in rather a conventional way. The four chapters on essentially potential flow cover almost half the course. It is notable that three-dimensional supersonic linearized theory is not taken up.

Chapter 8, Dynamics of a viscous liquid and gas. Viscous friction and dependence of viscosity on temperature are discussed. Navier-Stokes equations are derived and similarity concept and Reynolds number are introduced. Laminar flow in pipes and Stokes flow are discussed in detail. Shock structure is then discussed and there is an elaborate and well-written treatment of wakes and incompressible and compressible boundary-layer theory. Chapter 9, Turbulent motion. After a description of transition and a discussion of the averaging concept, author derives the Reynolds stress equation and presents the various similarity solutions (logarithmic, mixing length, etc.). He gives a good account of incompressible turbulent boundary layers and jets, and a brief introduction into modern research in isotropic turbulence.

Generally, this book contains a sound presentation of the main topics of fluid mechanics, with emphasis on classical theory and general theorems, and complete absence of supersonic wing theories. There is also a notable absence of references to recent western work.

Leon Trilling, USA

2081. **Krüger, H., On the influence of suction on the location of the transition point of wings** (in German), *Ing.-Arch.* **19,** 6, 384–387, 1951.

Laminar boundary-layer growth over two 15% thick profiles is calculated for the case of uniform continuous suction. A Joukowsky and a laminar profile at zero angle of attack are considered. Curves are presented for the laminar instability and separation points as functions of airfoil Reynolds number and suction velocity. Gerald E. Nitzberg, USA

2082. **Müller, W., On the longitudinal motion of a body of revolution in a fluid** (in German), *Ing.-Arch.* **19,** 4–5, 282–295, 1951.

The shape of a body of revolution at zero incidence and its associated pressure distribution are determined for a prescribed source-sink distribution along the axis. This source-sink distribution is represented in terms of Legendre polynomials. Reviewer notes that a similar method used by Young and Owen [*Aero. Res. Counc. Rep. Mem.* 2071, 1943] is not included in the list of references. G. M. Lilley, England

2083. **Roy, M., and Duban, P., Flow of a perfect liquid in a rotor with radial blades** (in French), *Bull. Assn. tech. marit. aéro.* no. 50, 145–159, 1951.

Using Spannhake's transformation $z^n = a(Z + 1/Z) + b$ (n = number of blades, a and b constants), the radial blades of the rotor in the z-plane are transformed into a single circle in the Z-plane.

The relative velocity field due to the circulation along the blades can be expressed as a Poisson integral in the Z-plane. Authors give some graphs of the tangential and radial velocity field of the circulation motion; also, some streamline patterns and pressure fields for different values of the transport through the rotor. H. G. Loos, Holland

2084. **Streeter, V. L., The ring doublet in ideal fluid flow,** Proc. Midwest. Conf. Fluid Dynamics, 1st conf., May 1950; J. W. Edwards, Ann Arbor, Mich., 56–65, 1951. **$10.**

A. Weinstein in 1948 [AMR **1,** Rev. 478] extended the applications of potential theory for body profile determinations to the usage of spatial source distributions, by properly interpreting Beltrami's integral of cylindrical functions for the potential of a ring, $2\pi\mu b\, {}_0\int^\infty e^{-zs}J_0(\rho s)J_0(bs)ds$, of radius b and strength μ/unit length derived by classical separation of variables for harmonic functions. Van Tuyl [AMR **3,** Rev. 1987] evaluated the associated stream potential and plotted the streamlines by transforming the similar integral into Jacobi elliptic integrals.

In contrast, Sadowsky and Sternberg obtained the same results directly [AMR **4,** Rev. 2099], by writing the Newtonian potential $1/R$ immediately in Jacobi elliptic functions and integrating around and across a ring for a disk distribution. In addition, they superimpose vortex rings and obtain longitudinal doublet distributions but plot no streamlines.

This work is paralleled by author's in a most lucid and straightforward paper, in which the streamlines are plotted for a distribution of doublets oriented longitudinally about a ring and superimposed on a uniform flow. Beginning from an excellent figure, the incremental doublets (used on the axis by von Kármán) are integrated around a ring for velocities, and the velocities across any curve $R^2/2ar + \cos\theta = \text{const}$ for the stream function. Bodies are generated from the particular solution at the stagnation point, a thick disk when on the axis and a multivalued stream function (torus) if off. Previously, the torus was obtained for a source ring by Weinstein. It is regrettable that comparison cannot be made with curves of the vorticity-circulation method of the more general third paper mentioned. Style continues author's clarity and engineering insight, although bodies are poor aerodynamically and of restricted usefulness in themselves. Rudolph L. Leutzinger, USA

2085. **Gilbarg, D., Unsteady flows with free boundaries,** *ZAMP* **3,** 1, 34–42, Jan. 1952.

Author considers a class of unsteady flows that is of much interest in connection with time-varying attached cavities in underwater motion of missiles. The problem is an extension of the known two-dimensional, steady-state cavity solutions to cases in which the condition of constant velocity magnitude on the free streamlines of the steady-state problem is replaced by the nonstationary form of Bernoulli's equation. The problem differs further in that the free boundary is a material line and not, in general, a streamline. The latter requirement is replaced by the approximating condition that the free boundary be a streamline and, thus, standard conformal mapping techniques are used to obtain solutions for cavities behind a flat plate normal to the flow direction (symmetric cavities). It is shown that these solutions are exact for unsteady flows whose free boundaries are of constant shape. In particular, two classes are distinguished: One in which the cavities have a cusped end or a stagnation point (the latter found by von Kármán [AMR **4,** Rev. 292] and included in the more general class of cavities of constant shapes found by author); the second yields cavities for which the free streamlines cross the axis of symmetry and are, thus, not physically realistic. In addition to the above detailed computations, method of extension of these results to arbitrary symmetric polygonal shapes and also to asymmetric polygonal shapes is indicated briefly. Phillip Eisenberg, USA

2086. **Funaioli, E., On the calculation of cascades of thin airfoils of nonnegligible curvature** (in Italian), *Pont. Acad. Sci. Acta* **14,** 45–55, 1951.

A method of calculation of flows through cascades of thin airfoils whose center lines approximate to circular arcs is given in principle, without details of computational procedures. M. J. Lighthill, England

2087. **Radok, J. R. M., The theory of aerofoils in unsteady motion,** *Aero. Quart.* **3,** part 4, 297–320, Feb. 1952.

Paper reviews work on two-dimensional, incompressible problems since 1936. Application to wing-tail combination is discussed. Good bibliography. John W. Miles, USA

2088. **Couchet, G., Aerodynamic forces acting on a profile in case of nonstationary motion with constant circulation** (in French), *C. R. Acad. Sci. Paris* **234,** 8, 808–810, Feb. 1952.

A supplementary note to the author's comprehensive work in *ONERA Publ.* 31 [AMR. **3,** Rev. 1308]. Gerald E. Nitzberg, USA

2089. **Lighthill, M. J., A new approach to thin aerofoil theory,** *Aero. Quart.* **3,** part 3, 193–210, Nov. 1951.

Author's general technique for rendering approximate solutions to physical problems uniformly valid is applied to the problem of correcting thin-airfoil theory near a rounded leading edge. Usefulness lies in the possibility of extension of theory to apply to lifting-surface problems. Method consists of expanding velocity field in power series in powers of a parameter defining size of disturbances; determining coefficients of the series by means of a lemma relating the velocity components at any point in the field to the velocity-distribution components, due to camber and thickness distribution, on the chord line; and investigating the nature

of the singularities in these coefficients. Author's previous work has shown that if there is a singularity near which successive terms in a series solution are each more singular than the last, so that series diverges near singularity, the divergence may be removed by straining the coordinate system. For problem studied, the strain transformation is a translation of velocity field downstream through a distance equal to half the leading edge radius of curvature, and new position of the singularity of the approximate velocity field is inside airfoil contour. Results are compared with various approximations given by Goldstein [AMR **1**, Rev. 882]. Author concludes with second-order approximations to lift, angle of zero lift, and airfoil surface-velocity distribution arising from angle of attack and from camber and thickness distributions.

Ellis Lapin, USA

2090. Kazakevich, F. P., Investigation of the drag of a circular cylinder in an air flow (in Russian), *Zh. tekh. Fiz.* **21**, 9, 1111–1120, Sept. 1951.

Pressure distributions as function of yaw ($0 \leq \theta \leq 60°$), and (precritical) Reynolds number ($0.4 \leq R \leq 1.3 \times 10^4$, based on diameter and full free-stream velocity), were obtained through four orifices (diam 0.3 mm) on a rotatable circular tube (diam d = 10 mm) in a 250 × 250-mm wind tunnel. Differential readings increased accuracy of normal-pressure drag measurements. The similarity of shapes of the distributions lead to a formula for normal drag in terms of differential pressure ($p_0 - p_{180}$) and yaw θ. Spanwise variation of drag was eliminated by placing two thin disks on the cylinder; since the results were insensitive to variation of distance b between the disks, $5d \leq b \leq 22d$, they were identified with ideal two-dimensional behavior. The resultant higher drag for $\theta = 0°$ then checked with CAGI Rep. no. 98, 1931, by Kuznetsov, where ratio of tunnel breadth w to diameter was $w/d = 100$. Further check was furnished by tubes of different diameters. As yaw increased, the spanwise "wall effect" diminished. Pressure orifices were also drilled perpendicularly to the elliptic sections of the yawed tube; the resultant drag differed by 2 – 3%, presumably the index of accuracy.

The trends of author's results with yaw check with the more extensive experiments by Bursnall and Loftin (where the wall effect, however, was present, w/d being 18); see AMR **5**, Rev. 814. Surprisingly, no mention was made of yaw theory, nor of turbulence level in the tunnel.

M. V. Morkovin, USA

2091. Williamson, J., The laws of flow in rough pipes (English and French), *Houille blanche* **6**, 5, 738–748, 749–757, Sept./Oct. 1951.

Author considers turbulent flow in rough pipes in the V^2-law region (drag coefficient independent of Reynolds number). Experience with three large concrete-lined aqueducts (diam 335, 412, and 610 cm, grain size k = 0.10 cm) showed that the pressure drop coefficient λ satisfied the formula $\lambda = 0.180\,(k/d)^{1/3}$ instead of $\lambda = (1.14 + 2 \log d/k)^{-2}$ (Nikuradse). Author shows also that Nikuradse's experimental results fulfill $\lambda = 0.180\,(k/d)^{1/3}$, provided that the influence of the lacquer coating (estimated to have a thickness of 0.0006 cm) on the effective height is taken into account. Therefore, the use of Manning's or Strickler's formula for the calculation of pipe friction is recommended. Author tries to base these formulas theoretically. Nikuradse's velocity profiles are represented by $V/V_m = Ax^n - Bx^m - C$ for $1.5\,k \leq x \leq 1$, in which V_m is maximal velocity, $x = 2a/d$, a distance from wall, $m = An/B$ (from $dV/dx = 0$ for $x = 1$), $C = -1 - B + A$ (from $V = V_m$ for $x = 1$). C varies from 0.18 for $d/k = 880$ to 0.32 for $d/k = 30$. Notwithstanding the high values of C, author sees the negative values of V for $x = 0$ as a reality and tries to explain this from the conception of the production of spinning masses near the wall, leaving the conception of a viscous liquid film.

Author simplifies too much the mechanism of production of turbulence causing drag, viz.: the work done by pressure = $\Delta p \cdot \frac{1}{4}\,\pi d^2 V_q$ (V_q is mean velocity) equals the work done in producing turbulence which is in ratio with $\pi d V_D{}^3$ (V_D is a velocity connected with drag; from Nikuradse's velocity profiles V_D appears to be the velocity at $x = 1.5\,k$, as for this distance $V_D \sim (\text{drag})^{1/3}$ for varying d/k). So $\lambda \sim (V_D/V_q)^3$. From Nikuradse's experiments it follows that $V_D/V_q \sim (k/d)^{1/9}$. These formulas lead to the Manning and Strickler formulas. From this theory, though based on straight pipe measurements, author explains the high resistance in pipe bends.

H. Wijker, Holland

2092. Garabedian, P. R., and Royden, H. L., A remark on cavitational flow, *Proc. nat. Acad. Sci. Wash.* **38**, 1, 57–61, Jan. 1952.

Authors present ". . . an alternate proof for the existence of plane cavitational flow . . . based on an extremal property of free streamlines developed formally by Schiffer. . ." The source reference for Schiffer's work is not cited. The physical interpretation of this extremal property is not made clear, and since both cavity and body have been taken as symmetrical in both x and y, it is difficult to see how results obtained here can be applied directly to a physical case.

L. Talbot, USA

2093. Casal, P., On the kinetic energy of a flow with a discontinuity surface of velocity (in French), *C. R. Acad. Sci. Paris* **234**, 8, 804–806, Feb. 1952.

Author aims to prove the theorem: If the steady flow of an inviscid incompressible fluid around an obstacle S possesses a surface of tangential velocity discontinuity Σ, and if the kinetic energy is a minimum with regard to any other flow having the same conditions at infinity but with a surface of discontinuity Σ', near to Σ, this flow is irrotational outside Σ, and Σ is a surface of equal pressure.

Proof is not clear to reviewer. Author appears to prove that the speeds of the fluid on Σ at either side of it are equal, from which he deduces that Σ is a surface of equal pressure.

J. C. Cooke, Malaya

2094. Rose, H. E., Breakdown of the Lambert-Beer law, *Nature* **169**, 4294, 287–288, Feb. 1952.

The Lambert-Beer law for the absorption of light has been verified experimentally for suspensions by measuring the intensity of the transmitted light as a function of the concentration of the suspension. It is found that the Lambert-Beer law is very accurate if no scattered light reaches the photosensitive surface of the recording apparatus. This is the case for powders of low reflectivity and if a small exit stop is used.

H. C. Brinkman, Indonesia

2095. Hasimoto, H., Note on Rayleigh's problem for a bent flat plate, *J. phys. Soc. Japan* **6**, 5, 400–401, 1951.

The author considers the same problem as was recently treated by Sowerby [AMR **4**, Rev. 3603]; namely, the flow starting from rest of a viscous incompressible fluid bounded by two infinite intersecting planes making an angle α which suddenly start to move with constant velocity W. However, whereas Sowerby considered only $\alpha = \pi/n$, n a positive integer, the author allows $0 < \alpha \leqq 2\pi$. The local friction coefficient is obtained as a series involving confluent hypergeometric functions, and the friction force per unit length is given explicitly for various multiples of $\pi/4$ and $\pi/6$ between $\pi/2$ and 2π. The results agree, as the

author points out, with Sowerby's where they overlap. The method is only sketched. J. V. Wehausen, USA

2096. **Moisil, Gr. C., Method of functions of hypercomplex variables in plane aerodynamics of incompressible viscous liquids** (in Romanian), *Acad. Repub. Pop. Române Stud. Cerc. Mat.* **1**, p 39, 1950.

It is well known that, in the theory of steady, incompressible, irrotational, plane flow of an ideal fluid, analytic functions $u(x, y) - iv(x, y)$ of the complex variable $z = x + iy$, where $i^2 + 1 = 0$, play an important role, inasmuch as any such function yields a possible flow of this kind, with u and v the velocity components in the x and y directions. L. Sobrero [*Mem. Atti Accad. Italia Mem. Cl. Sci. Fis. Mat. Nat.* **6**, 1–64, 1934; "Theorie der ebenen Elastizität unter Benutzung eines Systems hyperkomplexer Zahlen," Teubner, Leipzig, 1934] has shown a similar connection between the theory of plane elasticity and the theory of analytic functions $a(x, y) + jb(x, y) + j^2c(x, y) + j^3d(x, y)$ of the hypercomplex variable $x + jy$, where $(1 + j^2)^2 = 0$. The author shows a similar connection between analytic functions in the sense of Sobrero and the theory of slow, steady, incompressible, plane flow of a viscous fluid. For example, any analytic function of $x + jy$

$$F = -\Psi + j(\Phi - 4\mu v) + j^2(\Psi + 4\mu u) - j^3\Phi$$

represents such a plane flow, with body forces absent; and

$$\frac{dF}{dz} = \sigma_y + j(\tau - 2\mu\omega) - j^2\sigma_x + j^3\tau$$

where u and v are the velocity components in the x and y directions; σ_x, σ_y, τ are the stress components; μ is the coefficient of viscosity; and $\omega = \partial v/\partial x - \partial u/\partial y$ is the vorticity. Several examples of plane viscous flows are worked out in detail.

Courtesy of Mathematical Reviews J. B. Diaz, USA

2097. **Purday, H. F. P., An introduction to the mechanics of viscous flow. Film lubrication, the flow of heat by conduction, and heat transfer by convection,** New York, Dover Publications, Inc., 1949, 185 pp. **$1.25.**

This small volume, originally published in England under title "Streamline flow," treats viscous flow and heat-transfer problems pertinent to field of interest of author, who is chief designer for British diesel-engine firm. As subtitle indicates, consideration of viscous flow is limited almost exclusively to film lubrication, and a rather complete discussion of sliding pads and journal bearings is included, using Reynolds equation and dimensional analysis. Besides viscous flow in thin films, author discusses pipe and channel flow, making use of analogies to torsion and membrane problems, and flow about various obstacles, wherein results of Stokes, Blasius, etc., are quoted. Flow in porous media is also mentioned briefly. The following chapters give a brief account of heat transfer by conduction and forced convection, both boundary convection and heat transfer in a pipe being considered. The book concludes with a brief introduction to orthogonal functions, numerical solution of differential equations, and Navier-Stokes equations. Book also includes numerous examples worked out in great detail, a bibliography, and a list of symbols and units used.

Book is definitely written from standpoint of the practicing engineer and, at every opportunity, author avails himself of any approximations, which seem appropriate to particular problem in hand, to obtain useful results. This is indeed necessary in more difficult lubrication problems wherein cavitation and marked temperature effects may occur, for example.

Book should prove most useful to those in fields similar to that of author, although it is also recommended to anyone who seeks new problems in viscous fluid flow. The equations pertinent in lubrication problems have also been applied successfully to forming processes involving viscous fluids, as, for example, the pressing of molten glass. John A. Lewis, USA

2098. **Küchemann, D., and Weber, J., Concerning the flow about ring-shaped cowlings. Part IV. Further measurements on inlet devices,** *NACA TM* 1327, 21 pp., Dec. 1951.

Translation of *ZWB Forschungsber.* 1236/6, Mar. 1942.

2099. **Bory, C., Thermal convection with use of a movable wall** (in French), *C. R. Acad. Sci. Paris* **233**, 6, 471–473, Aug. 1951.

Author uses the boundary-layer velocity distribution he has obtained previously [AMR **5**, Rev. 773], for a semi-infinite flat plate lying parallel to a uniform stream; the plate being fixed for $x < l$ and moving parallel to itself with the speed of the free stream for $x > l$, to obtain the temperature distribution in the boundary layer if plate is at temperature of free stream for $x < l$ and at zero temperature for $x > l$. Problem is solved by dropping various terms from the governing differential equations in Lagrangian form, and author remarks that this procedure applied to previously computed boundary-layer problems yields numerical results in agreement to within 20% of the more accurate values. Peter Chiarulli, USA

2100. **Küchemann, D., Concerning the flow about ring-shaped cowlings of finite thickness. Part I,** *NACA TM* 1325, 24 pp., Jan. 1952.

Translation of *ZWB Forschungsber.* 1236, June 1940.

2101. **Küchemann, D., and Weber, J., Concerning the flow about ring-shaped cowlings. Part VIII—Further measurements on annular profiles. Part IX—The influence of oblique oncoming flow in the incremental velocities and airforces at the front part of circular cowls,** *NACA TM* 1328, 11 pp.; 1329, 16 pp., Feb. 1952.

Translation from *ZWB Forschungsber.* 1236/8, Mar. 1943, and 1236/9, June 1943.

Compressible Flow, Gas Dynamics

(See also Revs. 1982, 2080, 2089, 2139, 2159, 2162, 2166, 2167, 2168, 2200, 2201)

2102. **Burgers, J. M., Nonuniform propagation of shock waves. Mimeographed lecture notes prepared by S. I. Pai,** Inst. Fluid Dynam. appl. Math., Univ. Maryland, 1951, ii + 65 pp. $1.40.

In chapter I, several cases of the propagation of a one-dimensional plane shock in the gravitational field are treated, and the methods of integration of the equations of motion are outlined. In chapter II, the problem is generalized to include the symmetrical cylindrical and spherical waves. The last chapter deals with the motion of a gas in a gravitational field. Specifically, the problem is as follows: An infinite column of gas is bounded at its upper end by a plane and extends indefinitely in the downward direction. Initially, the gas is at rest, the pressure inside varies according to the law governing the distribution of pressure in an atmosphere, and the pressure on the upper side of the plane is zero everywhere. If sidewise motion is presented, what is the motion in the vertical direction after the bounding plane is suddenly removed? Both the formal solution and its interpretation are given. Y. H. Kuo, USA

2103. Nicholl, C. I. H., The head-on collision of shock and rarefaction waves, *Inst. Aerophys. Univ. Toronto UTIA Rep.* 10, 28 pp., 15 figs., 5 tables, Oct. 1951.

In a tube of constant cross section, a shock wave moves in one direction, and a rarefaction wave in the other. Paper investigates theoretically by the method of characteristics and experimentally by means of the shock tube the interaction of these waves. The experimental techniques, especially a piezoelectric pressure gage developed for this purpose, are described in detail. Tables are given which allow the variation of specific heats to be taken into account in the method of characteristics. The experimental values for the pressures after the shock has passed through the rarefaction are appreciably lower than the theoretical values.

Gottfried Guderley, USA

2104. Shapiro, A. H., Turbulent transfer processes in parallel jets, "Hydrodynamics in modern Technol.," Hydrod. Lab., Mass. Inst. Technol., 141–145, 1951.

Spark shadowgraphs and probe traverses have been made to investigate the transports of momentum, mass, and temperature in concentric coaxial gas streams moving at different speeds.

The rates of diffusion of the three transport properties have been obtained, showing the influence of velocity ratio and density ratio of the two streams and initial turbulence.

From author's summary by A. Petroff, USA

2105. Mathur, P. N., On the solution of Chaplygin's equation by means of Kummer's formula, Proc. Midwest. Conf. Fluid Dynamics, 1st Conf., May 1950; J. W. Edwards, Ann Arbor, Mich., 99–108, 1951. $10.

Author approximates pressure-density relation in subsonic, inviscid, compressible plane flow by a polynomial and reduces the hodograph equation to a confluent hypergeometric equation. No results are given.

Robert E. Street, USA

2106. Vincenti, W. G., and Wagoner, C. B., Transonic flow past a wedge profile with detached bow wave—details of analysis, *NACA TN* 2588, 48 pp., Dec. 1951.

Mathematical details of calculations of flow with detached bow wave past a doubly symmetrical, double-wedge profile at zero angle of attack are presented to supplement the description given in *TN* 2339 [AMR **4,** Rev. 4530]. Solution is based on transonic small-disturbance theory. Mixed-flow region is solved in the hodograph plane using finite-difference equations and relaxation techniques. Pure supersonic region is determined by the method of characteristics. A novel approach to finite difference solutions in the neighborhood of curved boundaries is indicated for the region adjacent to the shock polar. The material is presented in a systematic and clear manner.

Lester L. Cronvich, USA

2107. Germain, P., and Fenain, M., On a simple correspondence between the solutions of two partial differential equations, and its application to the approximate study of transonic flow (in French), *C. R. Acad. Sci. Paris* **234,** 6, 592–594, Feb. 1952.

By a suitable transformation, the equation

$$\psi_{\sigma\sigma} + a\sigma\psi_{\theta\theta}/(\sigma + b)^5 = 0 \qquad [1]$$

may be reduced to Tricomi's equation, and by suitable choice of constants a and b it may be used as a close approximation in the transonic range of velocities to Chaplygin's exact equation for flow in the hodograph plane. Numerical examples for a jet and for a nozzle are briefly reported. Reviewer notes that Eq. [1] has also been discussed by C. Loewner [*NACA TN* 2065, 1950].

D. C. Pack, Scotland

2108. Sacks, A. H., Behavior of vortex system behind cruciform wings—motions of fully rolled-up vortices, *NACA TN* 2605, 40 pp., Jan. 1952.

A detailed analysis is made of the motions of four fully rolled-up vortices trailing behind a cruciform wing banked 45° throughout the Mach number range. Equations are developed for the three-dimensional paths of the four vortices, and calculations are made of the distance behind the wing at which the upper two vortices pass ("leapfrog") through the lower two. These calculations are confined to wings of triangular planform.

It is found that the leapfrog distance decreases with increasing lift coefficient C_L and increases with increasing aspect ratio A. For angles of attack up to a certain critical value of $C_L/\pi A^2$, the vortex motion downstream is periodic, while above the critical value the motion is aperiodic. For low-aspect-ratio cruciform wings, the leapfrog phenomenon may occur within two chord lengths of the trailing edges.

The various types of vortex motion to be expected throughout the angle-of-attack range are considered in some detail, and the interaction of the two vortex sheets shed from the cruciform wing is taken into account. These theoretical results are compared with experiments in a water tank by visual-flow methods.

For angles of bank other than the special value 45°, it seems doubtful, due to the lack of symmetry, that calculations of this kind could be made by other than numerical procedures.

Manfred Schaefer, Germany

2109. Gontier, G., Contribution to the experimental study of the boundary layer in the neighborhood of sonic velocity (in French), *Rech. aéro.* no. 24, 3–8, Nov.-Dec. 1951.

Experimental investigations of the thickness and the velocity distribution of the boundary layer at Mach numbers 0.825 to 1.10 were conducted at the wind tunnel of the Institute of Fluid Mechanics at Lille (France). Using deformable walls of polished aluminum and observing all precautions, author could sustain a flow at Mach number 0.99 over a length of 180 mm. The static pressure was measured by small holes drilled into the wall, the dynamic pressure by a special Pitot tube at different distances from the wall. Plotting the results in semi-logarithmic coordinates, one obtains straight lines, which means a good confirmation of the logarithmic law given by von Kármán for incompressible turbulent fluid. The experimental results show some divergence only for the additive constant involved in this law. Experiments show further that, in the neighborhood of the critical sound velocity, the coefficient of friction may have a maximum, but for a final decision more data are necessary.

Karl Pohlhausen, USA

2110. Nocilla, S., On the problem of a delta wing with supersonic velocity (in Italian), *Monogr. Lab. Aero. Polit. Torino*, 13 pp. = *Atti. Accad. Sci. Torino* **85,** 1950–1951.

Treatment of the yawing delta wing by the standard method of conical flow (see also AMR **2,** Rev. 223, and Hayes, Browne and Lew, North American Report NA-46-818).

Gottfried Guderley, USA

2111. Orman, P. L., Rae, R. S., and Ward, G. N., Wind-tunnel test of a wing of finite aspect ratio of symmetrical double wedge section at supersonic speeds, *Proc. roy. Soc. Lond.* (A) **209,** 1098, 309–324, Nov. 1951.

Tests were made to find the characteristics of rectangular wings of such low aspect ratios that the linear theory for finite wings would not be expected to apply, and of such thickness that linear theory would fail at low Mach numbers. The wings, of aspect ratios 1.5, 1.25, 1.0, and 0.75, were 10% thick and were tested to

an incidence of 18° at Mach numbers of 2.3 and 1.5 and to an incidence of 9° at a Mach number of 1.3. Lift, wave drag, and normal force coefficients are given and compared with the corrected first-order theory of Schlichting for wings of finite aspect ratio. Agreement between theory and experiment is fair. Also included are tables of measured position of center of pressure in fraction of chord from the leading edge and in fraction of span from the tunnel wall to which the wing was fastened. Paper was written in 1946.

Neal Tetervin, USA

2112. **Miles, J. W., On Chang's function for nonstationary flow,** *J. aero. Sci.* **19,** 2, p. 138, Feb. 1952.

Author proves that Chang's function $C(\beta,M)$ [see AMR **4,** Rev. 4514] can be expressed in terms of Schwarz's function f_0 (λ,M).

Ed.

2113. **Vincenti, W. G., Comparison between theory and experiment for wings at supersonic speeds,** *NACA Rep.* 1033, 11 pp., 1951.

See AMR **4,** Rev. 2116.

2114. **Christophe, J., Computation of drag or thrust from surveys of wake or jet** (in French), *Rech. aéro.* no. 24, 9–15, Nov.-Dec. 1951.

Betz proposed a method of determination of drag of an airfoil in incompressible fluid by means of a survey of the wake. There are various formulas used to this end, notably that of Jones, etc. After reviewing some extensions of the method to compressible fluids (Gabaud, Lock, Hilton, Goldstein, Block, Katzoff, etc.), author proposes a general method for γ being constant or not. In the first case, the result is presented in form of equation. In the second case, the results are in form of diagram. Entropy is assumed to be a function of temperature only, which is justified by moderate pressures around the profile. Method is applicable to subsonic and slightly supersonic flows provided one neglects the shock influence in front of Pitot tube. Author maintains that his simple method is accurate enough for engineering purposes to determine quickly the drag of an airfoil or the thrust of a jet. Mathematics used is very elementary.

M. Z. Krzywoblocki, USA

2115. **Kirkby, S., and Robinson, A., Interference on a wing due to a body at supersonic speeds,** *Aero. Res. Counc. Lond. Rep. Mem.* 2500, 10 pp., Apr. 1947, published 1952.

See AMR **2,** Rev. 1526.

2116. **Blank, Helga, Application of Ritz's method to the calculation of compressible flow around a circular cylinder** (in Italian), *Monogr. sci. Aero.* no. 11, 11 pp., Nov. 1951.

The method of Rayleigh-Ritz is applied to problems in two-dimensional compressible flow. For this purpose the equation of potential compressible flow is replaced by a variational principle due to G. Braun. This is satisfied approximately by a suitable linear combination of functions which satisfy the boundary conditions individually. The method involves the use of a particular set of orthogonal functions of which only the first three are employed in practice. Mixed subsonic-supersonic flow around a circular cylinder for $\gamma = 1.5$ is calculated as an example. The resultant field of flow is symmetrical with respect to a diameter normal to the direction of flow. It appears to the reviewer that this, and the absence of shock waves in the solution, indicate that somewhere in the supersonic "bubble" the method ceases to be reliable.

(Mention may be made of related work by C. T. Wang—see *Proc. Internat. Congr. Mathematicians*, 1950, containing further references.)

A. Robinson, Canada

2117. **Chapman, D. R., Wimbrow, W. R., and Kester, R. H. Experimental investigation of base pressure on blunt-trailing-edge wings at supersonic velocities,** *NACA TN* 2611, 53 pp., Jan. 1952.

Results are given for 29 blunt trailing-edge wings with an aspect ratio of 3.0. The profiles vary in thickness ratio between 0.05 and 0.1, in boattail angles between −2.9° and 20°, and in ratio of trailing-edge thickness to airfoil thickness between 0.2 and 1.0. The range of Mach number lies between 1.25 and 3.1 and of Reynolds number between 0.2×10^6 and 3.5×10^6. Measurements on each wing were obtained separately with turbulent boundary layer and with laminar boundary layer.

Besides results of the mean base pressure as a function of the different profile and flow parameters, this report contains results of the spanwise distribution of the base pressure, the effects of various supports, a short investigation of various boundary-layer trips; furthermore, some schlieren pictures and some instructive china-clay photographs are included. Some exceptional cases involving laminar flow to the trailing edge gave unusually large effects of the different parameters, among others, discontinuities and hystereses in the slope. These effects are investigated and discussed in much detail.

As far as reviewer knows, this report presents the most complete and systematic investigation of base pressure on wings which exists.

S. F. Erdmann, Sweden

2118. **Warren, C. H. E., Dudley, R. E., and Herbert, P. J., A theoretical and experimental investigation of the flow in a duct of varying cross section, with particular application to the design of ducts for free-flight ground-launched model tests,** *Aero. Res. Counc. Lond. curr. Pap.* 60, 25 pp., 12 figs., Aug. 1950, published 1951.

Design of free-flight models of planes having jet engines must allow for internal flow through a duct for which the drag can be estimated. This drag is then subtracted from the measured total drag to give the required external-surface drag. Calculations are made on a duct having a straight section followed by a throat. Entrance conditions are $M = 0.4$ and Re, based on entry diameter, of 0.70 million. This corresponds to sea-level flight at $M = 1.1$. One-dimensional flow is assumed with a turbulent boundary layer starting at the entrance. Three ducts are designed by numerical solution of flow equations working stepwise. The third duct, giving a static pressure drop of 4.4:1, is judged suitable for producing no internal shocks over most of the contemplated speed range. Wind-tunnel tests on this duct verify the calculated pressure distribution and agree with one-dimensional channel theory for back pressures high enough to produce shocks in the channel. Finally, the internal drag is calculated as a function of M. For subsonic flight, some uncertainty arises because of insufficient data on base pressures.

Wayland Griffith, USA

2119. **Fabri, J., Siestrunck, R., and Fouré, C., Choking phenomena associated with flame stabilization** (in French), *Rech. aéro.* no. 25, 21–27, Jan.-Feb. 1952.

Authors treat steady-state two-dimensional flow field in straight duct in which combustion of homogeneous gaseous mixture is stabilized by a small obstacle located on axis of symmetry. Assumption of constant ratio of stagnation temperatures across flame front, made in Tsien's treatment of the problem [AMR **4,** Rev. 3985], is criticized and replaced by assumption of constant ratio of stagnation enthalpies.

Flow field is computed from relation between vorticity in burned gas and entropy jump across flame front, neglecting small terms such as squares of velocity component normal to flame front. Velocity profiles and shapes of flame front are given for several values of approach stream velocity u_0 and stagnation enthalpy ratio λ.

Similar to Tsien's results, authors find that flame spreading to wall is incompatible with approach velocities in excess of a value u_0^* that decreases with increasing λ. They state that this value is identical with that derived on basis of one-dimensional theory. Unlike Tsien, authors seem to be aware that the only valid conclusion to be drawn from this result is: Amount of heat release imposes upper limit on u_0, which in any given system adjusts spontaneously to compatible value, depending on boundary conditions [cf. Foa and Rudinger, AMR **2**, Rev. 507].

George H. Markstein, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 2081, 2099, 2109, 2133, 2172)

2120. **Watson, E. J., and Preston, J. H., An approximate solution of two flat plate boundary-layer problems,** *Aero. Res. Counc. Lond. Rep. Mem.* 2537, 13 pp., Aug. 1946, published 1951.

An approximate method is given for Blasius' solution of boundary-layer flow past a flat plate. The method is applied to a uniform stream with a suction velocity normal to the plate and inversely proportional to the square root of the distance from the leading edge. An initial approximation to the velocity is made using (a) a constant value $f_1'(\eta) = K$, $[\eta = (U/\nu x)^{1/2}y;\ u = Uf'(\eta)]$; (b) a linear function $f_1'(\eta) = K\eta$. After the third-order nonlinear differential equation is solved by iteration, a second approximation containing K is found. This parameter is determined using the momentum equation.

Methods (a) and (b) were applied to the flow past a flat plate, and (a) was applied to the case with suction velocity. The quantities τ_0, δ^*, θ, and H were determined as functions of the suction. Authors consider as one of the advantages of this method that it covers a larger range of suction than other methods.

F. N. Frenkiel, USA

2121. **Kuethe, A. M., Some aspects of boundary-layer transition and flow separation on cylinders in yaw,** Proc. Midwest. Conf. Fluid Dynamics, 1st Conf., May 1950; J. W. Edwards, Ann Arbor, Mich., 44–55, 1951. $10.

The influence of yaw on the stability of laminar boundary layers is investigated theoretically and experimentally. In the flow equations (three-dimensional momentum, continuity, and energy), the spanwise derivatives are usually neglected. Author investigates the effects of this neglect and states that the flow in the chordwise direction and perpendicular to it is affected by yaw, and, particularly for turbulent phenomena, the spanwise derivatives should not be neglected. Three-dimensional disturbances are more important in the yawed than in the unyawed case. The chordwise and spanwise profiles in the boundary layer have different stability limits to a three-dimensional disturbance. The fully developed laminar flow in a channel of breadth $2b$ is considered, and the effect of three-dimensional disturbances is studied. The equations are linearized by assuming that the disturbance velocites are small, and higher than first-order terms are neglected. It is found that the rate of amplification of a three-dimensional disturbance is the same as that of a two-dimensional one at a lower value of the Reynolds number and a higher wave number. In particular, results of two- and three-dimensional analyses of the boundary layer of a flat plate are compared. The results of pressure-distribution tests on a circular cylinder at various angles of yaw through the critical Reynolds number range are presented and discussed. Author states that check tests were being made in the NACA low-turbulence wind tunnel, since no low-turbulence wind tunnel was available for the original work.

Author concludes that, although the linearized disturbance theory is not suitable for describing boundary-layer transition, the results of the analysis imply that the effect of yaw is to move boundary-layer transition forward in an airstream of low turbulence. The tests on the circular cylinder indicate that the effect of yaw is to move the transition point upstream.

Paul Torda, USA

2122. **Deissler, R. G., and Eian, C. S., Analytical and experimental investigation of fully developed turbulent flow of air in a smooth tube with heat transfer with variable fluid properties,** *NACA TN* 2629, 43 pp., Feb. 1952.

Measurements were taken of local coefficients of heat transfer and friction for flow of air in an electrically heated tube (diam 0.87 in., length 87 in.) at high ratios of wall-to-fluid bulk temperatures. In certain cases velocity and temperature profiles were also measured. Main aim was to verify an analysis expounded in previous reports [AMR **4**, Revs. 1699, 3012]. The analysis distinguishes a wall region, where molecular and eddy diffusivities ($\epsilon = n^2uy$, where $n = 0.109$ is an empirical constant, u velocity at distance y from wall) are both considered, and a region at distance from wall, where only eddy diffusivity is considered, for which von Kármán's similarity relation with $\kappa = 0.36$ is used. The analysis is further based on the assumptions: (1) Equal eddy diffusivities for momentum ϵ and for heat transfer ϵ_h; (2) uniform shear stress τ and heat flux q across tube; (3) constant static pressure across tube; (4) constant Prandtl number Pr and specific heat.

Comparison with their experimental results led authors to conclude that substantial agreement was obtained with predicted heat-transfer correlations, friction correlations, velocity distributions, and temperature distributions, at least for large Reynolds numbers ($Re > 15.000$).

For the region at distance from wall, assumption (2) seems very risky, but authors show by analysis that differences with a linear behavior of τ and q and y are small and only noticeable for $y >$ half tube radius; even then the differences are scarcely greater than experimental accuracy.

Reviewer remarks that neither is assumption (1) in agreement with experimental evidence, which shows $\epsilon_h/\epsilon = 1.4 - 1.5$ [Woertz and Sherwood, *Trans. A. I. Chem. E.* **35**, p. 517, 1939; Reichardt, *ZAMM* **20**, p. 297, 1940]. However, it can be shown that the same analytical results for the region at distance from wall will be obtained if it is assumed that $(\epsilon_h/\epsilon)(\tau/\tau_0)(q_0/q) \simeq 1$ (subscript 0 refers to wall). From Reichardt's paper it may be concluded that for $Pr = 0.72$ and $Re \simeq 10^5$, values pertinent to Deissler's experiments, the above product is indeed nearly equal to 1. Furthermore, for $Re > 15.000$, the measured temperature distributions are slightly flatter than those predicted by authors' analysis.

J. O. Hinze, Holland

2123. **Townsend, A. A., On the fine-scale structure of turbulence,** *Proc. roy. Soc. Lond.* (*A*) **208**, 1095, 534–542, Sept. 1951.

Starting from the consideration of the discrepancy existing between experimental results and some consequences of the hypothesis of similarity of turbulent processes during decay, author first introduces, instead of the parameters kinematic viscosity and total energy dissipation, the equivalent set of parameters kinematic viscosity and mean square distortion rate, as the controlling factors in the similarity range. He then proceeds to formulate a

new hypothesis concerning the nature and origin of the smallest eddies of turbulence, which are assumed to be originated by perturbations of vorticity which, under the action of the distortion due to the main vorticity field, are concentrated into vortex sheets or lines. It is seen at once that this turbulence pattern complies with the general requirements indicated by theory and experiments for the smallest eddies, as far as factors governing the motion, characters of the spectrum function, balance between energy supply and dissipation, and flattening factor of $\partial^3 u/\partial x^3$ are concerned.

To corroborate his hypothesis, author computes the spectrum functions due to two particular types of motion, namely, a plane shearing motion: $u = \alpha x$, $v = -\alpha y$, $w = 0\ (\alpha > 0)$, which has a solution representing a vortex sheet of finite thickness; and an axial straining: $u = 2\beta x$, $v = -\beta y$, $w = -\beta z\ (\beta > 0)$, whose solution represents a vortex line of finite thickness. The spectra obtained in this way, particularly the vortex sheet spectrum, agree quite well with experimental results at very large wave numbers. The conclusion, based also on independent evidence by Burgers, is that the representation of the smallest eddies as a random distribution of vortex sheets closely approximates the real conditions. Considering the reasons for this result, author points out causes of inconsistency of some results obtained by applying Heisenberg's energy-transfer hypothesis and discusses limits of validity of this latter hypothesis. P. L. Romita, Italy

2124. **Taylor, J. F., and Comings, E. W., Impact-tube measurements in isothermal air jets,** Proc. Midwest. Conf. Fluid Dynam., 1st Conf. May 1950; J. W. Edwards, Ann Arbor, Mich., 204–215, 1951. $10.

Effect of turbulence in jet of about 180-fps velocity is investigated by impact-tube traverses. By use of screen and varying nozzle throat lengths, turbulence level (defined as square root of ratio of perturbation velocity squared to mean velocity squared) varied from zero to 8%. In region where jet does not diverge, turbulence has some effect on velocity distribution near nozzle, but effect disappears at distance of 12 times nozzle diameter. No evidence is found that turbulence affects spread of jet. Reading of impact tube is not sensitive to deviations of angle of orientation up to 15 deg from normal. In diverging portion of jet, effect of turbulence may be marked under certain conditions, but this study is not carried to conclusion. C. W. Smith, USA

2125. **Giedt, W. H., Effect of turbulence level of incident air stream on local heat transfer and skin friction on a cylinder,** *J. aero. Sci.* 18, 11, 725–730, 766, Nov. 1951.

Data are presented for local heat-transfer coefficient, pressure coefficient and skin-friction coefficient on a cylinder below and in the critical range of Reynolds number. The effect of artificially increased turbulence in the free airstream on these local quantities was also determined. Data were first taken with a 16-mesh damping screen (less than 1% resulting turbulence level) placed upstream of the test section and then with greatly increased turbulence (about 4%) obtained by using a 1.25 × 1.25-in. rope net of $\frac{3}{16}$-in. diam cord. Warren M. Rohsenow, USA

2126. **Chandrasekhar, S., The fluctuations of density in isotropic turbulence,** *Proc. roy. Soc. Lond.* (A) 210, 1100, 18–25, Dec. 1951.

The density correlation function $\omega(r, t) = \overline{\delta\rho\, \delta\rho'}$, where $\delta\rho$, $\delta\rho'$ are the fluctuations of density from the mean at points separated by a distance r, is defined for the isotropic turbulent motion of a compressible fluid. From the equation of continuity is deduced the invariant relation, $d/dt \int_0^\infty \omega(r, t)\, r^2\, dr = 0$, which may be interpreted similarly to the Loitsianskiĭ invariant relation. An equation for the propagation of ω is then deduced assuming normality of the joint probability distribution function for $\delta\rho$, $\delta\rho'$ and the velocity components and the adiabatic relation between pressure and density. Further assuming that the velocity fluctuations are small compared with the velocity of sound and that viscosity and conductivity may be neglected, it is found that ω satisfies the spherical wave equation

$$\frac{\partial^2 \omega}{\partial t^2} = \frac{2\left(c^2 + \frac{1}{3}\overline{u^2}\right)}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \omega}{\partial r}\right)$$

and the author concludes that $\omega(r, t)$ can be expressed as a superposition of spherical waves.

The reviewer is of the opinion that this solution represents an isotropic sound field not necessarily related to the turbulence. Since the assumption of normality of the joint probability distribution function equates to zero that inertial transfer of turbulent energy which is characteristic of turbulent motion, it is unlikely that a correct account of the density field can be obtained in this way. A. A. Townsend, England

2127. **Coburn, N., A method for constructing correlation tensors in homogeneous turbulence,** Proc. Midwest. Conf. Fluid Dynamics, 1st conf., May 1950; J. W. Edwards, Ann Arbor, Mich., 129–141, 1951. $10.

The usual Kármán-Howarth theory is reframed by decomposing the various tensors along three independent unit vector fields and then requiring invariance of these quantities under specific groups of transformations of the unit vector fields. Several types of symmetries are examined and compared with the usual homogeneous isotropic case, where invariance is required under parallel displacements and under orthogonal transformation of the plane perpendicular to the direction of correlation.

Albert F. Pillow, Australia

Aerodynamics of Flight; Wind Forces

(See also Revs. 1923, 2012, 2087, 2090, 2113, 2118, 2156)

2128. **Bensen, I. B., Performance data and operating experience of a pressure jet helicopter,** *Fairchild Publ. Fund., Inst. aero. Sci. Prepr.* 362, 26 pp., 14 figs., 1952.

Paper presents the results of an extensive evaluation study conducted on a pressure-jet helicopter, with particular emphasis on the thermodynamics of its power cycle. Operational experience, mechanical problems, and over-all performance of this type of helicopter jet-propulsion power plant are discussed.

Part of the intent of the evaluation was also to verify the existing theoretical methods for predicting performance of this type of helicopter prime mover.

On the basis of data obtained, the pressure-jet power cycle shows great promise as the most efficient jet cycle proposed to date, with a further advantage of low installed prime-mover weight and unusually large power reserve. Pressure-jet rotors are also quite insensitive to variations of tip speed; this permits their operation always at optimum lift condition.

From author's summary by W. Z. Stepniewski, USA

2129. **Walker, W. G., and Steiner, R., Summary of acceleration and airspeed data from commercial transport airplanes during the period from 1933 to 1945,** *NACA TN* 2625, 30 pp., Feb. 1952.

Normal acceleration and airspeed data collected with NACA *V-G* recorders in transport airline operations from 1933 to 1945 are summarized and analyzed with respect to gusts and gust loads. The accelerations experienced in most operations equaled or

exceeded the limit-gust-load-factor increment, on the average, twice (once positive and once negative) in 10^7 flight miles. The gusts experienced in most operations exceeded 33 fps, on the average, twice (once positive and once negative) in 10^7 flight miles. The loads experienced for several operations varied appreciably from average conditions. A predominating factor causing the variations in the load experience was the difference in the gust experience, with operating speeds in rough air being a secondary contributing or moderating factor.

From authors' summary

2130. Lazzarino, L., Consideration of the possibility of calculating certain design parameters of an airplane so as to produce prescribed longitudinal stability characteristics (in Italian), *Aerotecnica* **30**, 1, 25–31, Feb. 1950.

Equations and methods of solution are presented for solving the aircraft dynamic problem in reverse. Author makes use of the relationships existing between the coefficients of the characteristic or determinantal equation and the roots of this equation. On assigning values to the roots such that the desired stability characteristics are achieved, he proceeds to demonstrate how one may possibly extract the basic design parameters from the complex relationships which exist for the determinantal coefficients. Unfortunately, there is no way of guaranteeing that a solution can be obtained, i.e., real positive numbers whose magnitudes are sensible from a practical point of view. In the reviewer's opinion, this design task is best handled by solving the dynamic problem directly, avoiding the "extreme laboriousness" of the above-described computation. Experience has shown that analog computers are admirably suited for the solution of this problem, in which it is desired to vary design parameters over a wide range in order to obtain the desired response characteristics.

Leonard Segel, USA

2131. Brown, B. P., Chilton, R. G., and Whitten, J. B., Flight investigation of a mechanical feel device in an irreversible elevator control system of a large airplane, *NACA TN* 2496, 47 pp., Oct. 1951.

The longitudinal stability and control characteristics of a large airplane have been measured with a mechanical feel device in combination with a booster incorporated in the elevator-control system. Tests were made to investigate the feasibility of eliminating the aerodynamic control forces through use of a booster and providing control-feel forces mechanically. The feel device consisted of a centering spring which restrained the control stick through a linkage which was changed as a function of the dynamic pressure.

During the tests, the over-all performance of the feel device was satisfactory. The control effort of the pilot was completely dependent upon the feel-device setting, but the stick-fixed stability was not appreciably affected by the device. The stick-fixed characteristics of the airplane without the feel device, however, were satisfactory. The original conventional control system of the test airplane exhibited certain undesirable stick-force characteristics which resulted from nonlinear hinge-moment variations which were improved or corrected by the feel device. The feel device provided smoother landings with less pilot effort and improved the stick-force characteristics in maneuvers.

From authors' summary

2132. Chamouton, D., Lateral stabilization of airplanes (in French), *Bull. Assn. tech. marit. aéro.* no. 50, 191–213, 1951.

Although paper is mainly a summary of known results, it contains interesting material concerning (1) practical determination of stability derivatives, and (2) lateral stabilization by automatic pilot.

Author derives equations of lateral motion and explains experimental determination of stability and control derivatives by frequency response method (see AMR **3**, Rev. 963). Accuracy of this method is claimed to be superior to balance measurements. Next, he investigates necessary relation $\delta(\alpha)$ between rudder angle δ and aileron angle α to insure zero sideslip, and suggests how $\delta(\alpha)$ can be determined in flight by frequency-response method. Experimental results are discussed and importance of including product of inertia E when calculating maximum frequency is emphasized.

Reviewer has noted the following errors: $q = \frac{1}{2}\rho V^2$, dynamic pressure, p. 193. In rolling and yawing moment equations, p. 194, defining ΣM_x and ΣM_z terms containing C_{lp}, C_{lr}, C_{np}, C_{nr} should be divided by 4. Right-hand side of equation (1), p. 195, should be $+C_{l\alpha}\alpha$. For C_r read C_z in definition of Δ_3, p. 198.

Sidney Kirkby, England

2133. Dowlen, E. M., A shortened method for the calculation of aerofoil profile drag, *J. roy. aero. Soc.* **56**, 494, 109–116, Feb. 1952.

A very simple and rapid method is developed for calculating the profile drag of any airfoil with known velocity distribution and position of transition point. The momentum thickness of the boundary layer at the trailing edge is found by a universal integration of the momentum equation of the boundary layer due to N. Tetervin [*NACA ACR* L4G14, 1944]. A relation between the drag coefficient and the momentum thickness at the trailing edge is given from H. B. Squire and A. D. Young [*Aero. Res. Counc. Rep. Mem.* 1838, 1937]. By this means the drag coefficient can be obtained by two simple integrals evaluated from the given velocity distribution and some universal quantities represented in diagrams.

The reviewer remarks that a similar method has been given by H. B. Helmbold [AMR **3**, Rev. 2374]. His equation even avoids the velocity at the trailing edge [see also H. Schlichting, "Boundary-layer theory," AMR **5**, Rev. 194].

N. Scholz, Germany

2134. Smiley, R. F., A semiempirical procedure for computing the water-pressure distribution on flat and V-bottom prismatic surfaces during impact or planing, *NACA TN* 2583, 28 pp., Dec. 1951.

Simple equations are developed for rectangular flat plate by considering several previous theoretical derivations and experimental results, which show good agreement with experimental results for trims below 30° and wetted length-beam ratios up to 3.3. Development assumes longitudinal distribution of pressure is mainly function of the normal load coefficient so that it may be computed from existing two-dimensional flow theory. Transverse distribution is obtained as a compromise between existing theories for small and large wetted length-beam ratios. Experimental results show that a simple modification to the flat plate equations can be used to predict approximately the pressure on V-bottom surfaces.

From author's summary by A. F. W. Langford, Australia

2135. Hamilton, J. A., An investigation into the effect of forced and natural afterbody ventilation on the hydrodynamic characteristics of a small flying boat (Saro 37) with a 1:15 fairing over the main step, *Aero. Res. Counc. Lond. Rep. Mem.* 2463, 28 pp., Dec. 1946, published 1951.

Previous attempts to reduce the air drag of a flying-boat hull by means of a fairing over the main step have usually produced undesirable water-planing characteristics due to the suction force created on the afterbody. This investigation, conducted on a full-scale aircraft, indicates that by means of natural and forced

ventilation of the afterbody during taxiing, take-offs, and landings, the water performance of the aircraft can be measurably improved. A series of tests were conducted using various ventilation arrangements and the results of these tests were plotted to form characteristic curves of stable and unstable regions. Ventilation of the afterbody appears to reduce skipping considerably, although the improvement of forced ventilation over natural ventilation for the blower capacities tested was not too significant.

Robert S. Ross, USA

2136. **Owen, T. B., Draft corrections for water surface deflection under no. 1 carriage for the R.A.E. seaplane tank,** *Aero. Res. Counc. Lond. curr. Pap.* 67, 10 pp., 6 figs., Mar. 1950, published 1952.

The running draft and attitude of hulls under test are measured relative to the undisturbed water level, but at normal running speeds an appreciable deflection of the water surface is produced at the model testing position by the aerodynamic pressure field of the carriage. These deflections have been measured and data are given for the correction of measured draft and attitude values covering the normal range of model positions and conditions. When testing models are screened from air flow, there is an inclination of the surface of $-0.4°$ at 10 fps and a deflection of -0.23 in. at 12 fps. It is therefore recommended that planing tests on hulls should be confined to speeds above 16 fps, where the surface inclination is small and the deflection decreases steadily from -0.09 in. at 16 fps to -0.03 in. at 36 fps. When testing models in air flow, there is no appreciable inclination of the water surface, and the peak deflection of -0.06 in. at 12 fps decreases to -0.02 in. at 16 fps, and remains at this value up to 38 fps.

From author's summary

2137. **Monaghan, R. J., A review of the essentials of impact force theories for seaplanes and suggestions for approximate design formulae,** *Aero. Res. Counc. Lond. Rep. Mem.* 2720, 27 pp., Nov. 1947, published 1952.

Classical theories of impact of seaplanes on water have been based on the assumption of a transfer of momentum to a hypothetical associated mass of water attached to the seaplane, such that the total momentum of the two remains constant. Recent developments of the theory show that this treatment fails to take account of momentum shed to the wake formed behind a seaplane when it has forward speed, i.e., it neglects the planing forces.

This report reviews the essential theory and assumptions underlying recent work, and puts forward an approximate design formula for the maximum deceleration during a main step impact which is directly a function of the initial impact conditions. It has the form $(dV_n/dt)_{\max} = -A\,(K\rho/M)^{1/3}V_{no}^{\;2}$, where V_{no} is the velocity normal to the keel at first impact, the factor A is uniquely determined by the ratio of the flight path angle to the attitude, K is a function of the geometry and attitude of the step, which depends on the assumptions made in defining the associated mass, ρ is the density of the fluid, and M the mass of the seaplane. Values of the constants are given in generalized curves.

From author's summary

2138. **Nonweiler, T., Theoretical stability derivatives of a highly swept delta wing and slender body combination,** *Coll. Aero. Cranfield Rep.* 50, 31 pp., 5 tables, 17 figs., Nov. 1951.

Report utilizes linearized slender-body concept and the perturbation potential to calculate stability derivatives in two additive parts; one for the wing-body portion, and the other for the body-nose section ahead of the juncture of body and leading edge of the wing. The method is carried out for a delta wing symmetrically attached to the cylindrical portion of a slender body with arbitrary pointed nose. Tabulated results for the body-nose portion refer to a conical nose. Three force and three moment derivatives are given corresponding to changes in longitudinal speed, angle of attack, pitch, sideslip, yaw, and roll. Figures are given showing the effect of body diameter to wing-span ratio. A transformation table relates derivatives with respect to wind axes to those about body axes. Paper supplements and partially overlaps results of Spreiter [*NACA TN* 1662, 1948], Jones [AMR **4**, Rev. 2641], and Heaslet and Lomax [AMR **3**, Rev. 2748].

H. P. Liepman, USA

2139. **Lomax, H., and Byrd, P. F., Theoretical aerodynamic characteristics of a family of slender wing-tail-body combinations,** *NACA TN* 2554, 75 pp., Nov. 1951.

Authors study aerodynamics of sweptback wing plus triangular tail on cylindrical body when Mach number is near unity, or when entire configuration is slender. Potential equation then does not include axial term, i.e., slender-wing theory is used. Wing leading edge is assumed straight, trailing edge curved to yield flat loading curve outboard of body. Two tail-loading cases are considered: (1) Wing trailing vortex sheet flat and in tail surface plane; (2) trailing vortices roll up into two-point vortices in or out of tail surface plane.

This clearly presented study includes details of analysis, many illustrative sketches, and tables and graphs of results of variation of aerodynamic quantities with configuration.

Morton Finston, USA

2140. **Nonweiler, T., The theoretical lift and pitching moment of a highly-swept delta wing on a body of elliptic cross section,** *Aero. Res. Counc. Lond. curr. Pap.* 58, 11 pp., 5 figs., June 1950, published 1951.

This note investigates the lift, pitching moment, and induced drag coefficients of a highly swept delta wing attached to an elliptic cylinder of constant cross section. These coefficients are derived by treating the changes in perturbation velocity (produced by the body) parallel to the free-stream direction as small compared with the velocity changes in transverse planes.

The results are obtained by a conformal transformation of the flow in transverse planes about the delta wing plus circular body combination. The pressure difference between the upper and lower surfaces of the wing and body are obtained by using Bernouilli's equation for linearized flow.

Curves are given which enable these coefficients to be determined for various values of the body width and body height.

Ammon S. Andes, USA

2141. **Merriam, K. G., A note on wind-tunnel wall effect corrections for rectangular airfoils,** *J. aero. Sci.* **18**, 12, p. 845, Dec. 1951.

Note in Reader's Forum.

2142. **Lean, D., Flight tests on the Youngman-Baynes high-lift experimental aircraft,** *Aero. Res. Counc. Lond. curr. Pap.* 65, 28 pp., 26 figs., Aug. 1950, published 1952.

Flight tests on the Youngman-Baynes experimental high-lift aircraft have shown that an increment of maximum lift coefficient of 1.32 can be obtained on an unswept wing with a low drag section, whose basic maximum lift coefficient is 1.28. It is estimated that an increment of maximum lift coefficient of 0.2 has been lost due to the adverse effect of wing-fuselage interference.

Adequate lateral control in all conditions of flight is provided by ailerons inset in the full span flap.

The profile drag-coefficient increment at full flap is 0.07 for a lift-coefficient increment of 1.14, at a wing incidence of 10 degrees.

The changes in longitudinal trim due to the flaps are small and easily controlled, and the effect of ground on longitudinal trim is considered negligible, since there is no difficulty in landing with flaps down.

The structure of the wing-flap-aileron arrangement is adequately stiff in torsion, and the aileron reversal speed is estimated to be nearly 300 knots. From author's summary

2143. Cole, H. A., Jr., and Ganzer, V. M., Experimental investigation of rolling performance of straight and sweptback flexible wings with various ailerons, *NACA TN* 2563, 45 pp., Dec. 1951.

Tests were conducted in an 8 × 12-ft tunnel on two flexible wings, one with 45° sweepback and the other unswept. Rolling moments due to aileron deflection, damping derivatives in roll, and free-rolling angular velocities due to aileron deflection were obtained at various speeds including, when possible, the aileron reversal speed.

The following results were obtained: (1) When designed for equal stress, aileron reversal speeds were higher for the swept wing than for the straight wing; (2) aileron reversal speeds for a straight wing occurred at nearly the same speed with all percent-span ailerons but varied widely for the swept wing; (3) ailerons mounted inboard on the swept wing resulted in much higher reversal speeds than did the same percent-span ailerons mounted at the tips; (4) theoretical values obtainable for the straight wing checked experimental values quite closely. The agreement was not satisfactory for the swept wing, with the particular theory used overestimating the reversal speed in most cases.

Leonard Goland, USA

2144. Wolhart, W. D., Influence of wing and fuselage on the vertical-tail contribution to the low-speed rolling derivatives of midwing airplane models with 45° sweptback surfaces, *NACA TN* 2587, 55 pp., Dec. 1951.

One of a series of investigations being conducted by the NACA to determine experimentally how the variation of certain airplane components may influence the subsonic stability characteristics of other components. This paper shows the influence of the wing and fuselage on the vertical-tail contribution to the rolling derivative. Seymour Lampert, USA

2145. Mair, W. A., High-speed wind-tunnel tests on models of four single-engined fighters (Spitfire, Spiteful, Attacker and Mustang). Parts 1–5, *Aero. Res. Counc. Lond. Rep. Mem.* 2535, 79 pp., Apr. 1945, published 1951.

Report describes measurements of lift, drag, and pitching moment made in the R.A.E. High-Speed Wind Tunnel on models of the Spitfire, Spiteful (F.1/43), Attacker (E.10/44), and Mustang. On the Spiteful model, pressure distributions on the front radiator flap were also measured. An introduction (written in 1949) gives a general account of the tests described in the separate parts of the report. From author's summary

Aeroelasticity (Flutter, Divergence, etc.)

(See also Rev. 2158)

2146. Kinnaman, E. B., Flutter analysis of complex airplanes by experimental methods, *Fairchild Publ. Fund. Inst. aero. Sci. Prepr.* 363, 15 pp., 20 figs., 1952.

Qualitative description of Boeing's experience in aeroelastic model testing is presented. General information regarding value of data, experimental techniques, peculiar problems, and general tricks of the trade is provided. Several flexible mounting systems for pseudo free-flight wind-tunnel models are described. Model constructional techniques are explained. Photographs and diagrams clearly amplify the text.

This is a valuable contribution to experimental aeroelastic techniques which rarely appear in the formal literature.

C. Desmond Pengelley, USA

2147. Baird, E. F., Pines, S., and Winson, J., The solution of two problems of four degree of freedom flutter by electronic analogue computation, Reeves Instrum. Corp., New York, 59 pp., Feb. 1952.

This expository article describes in complete detail material of subject. See also a report by J. Winson [AMR **4**, Rev. 1738]. Article is well written and sufficiently clear so that similar problems can be easily set up and run on an analog computer. Aerodynamically, incompressible flow for an airfoil of constant chord and infinite aspect ratio is postulated. Y. Luke, USA

2148. Woolston, D. S., and Castile, G. E., Some effects of variations in several parameters including fluid density on the flutter speed of light uniform cantilever wings, *NACA TN* 2558, 40 pp., Dec. 1951.

Flutter-test results are presented for a number of unswept wings of aspect ratio 4, 6, and 8. The wings were tested in air and Freon-12 over a wide range of tunnel pressures. This permitted a large variation in the ratio of the wing mass to fluid density. A comparison of the experimental and calculated flutter speeds showed that the calculations were in reasonable agreement with the tests for normal to heavy wings, but that for very light wings there was a large difference between theory and experiment. The reason for these differences is not understood.

Arthur A. Regier, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Rev. 2077)

2149. Hilton, W. F., Supersonic propellers, *J. roy. aero. Soc.* **55**, 492, 751–761, Dec. 1951.

Author presents a summary of topics discussed in a lecture to the Royal Aeronautical Society on the efficiency, noise, thermal effects, power-plant design, and possible applications of the supersonic propeller. Some of the material is admittedly speculative; the structural problems associated with the thin-airfoil sections required for such a propeller have been avoided in order that the discussion may be on a topical basis.

The thermal barrier occurring at approximately $M = 5$ for steel projectiles and unpowered gliders is discussed in some detail; author makes use of this material in speculating on very high speed types of power plants.

Concluding remarks state that there appear to be no apparent theoretical or practical disadvantages to the supersonic propeller when applied in its proper sphere, but there appears to be little practical application for the transonic propeller. Reviewer is in complete accord with these statements.

James B. Duke, USA

2150. Prian, V. D., and Michel, D. J., An analysis of flow in rotating passage of large radial-inlet centrifugal compressor at tip speed of 700 feet per second, *NACA TN* 2584, 46 pp., Dec. 1951.

An analysis is made on a radial-inlet centrifugal compressor to provide more knowledge of the flow conditions within the impeller. The tests indicate a region of low efficiency generally existing along the trailing face of the blades as a result of both shifting

low energy air toward the trailing face and due to losses arising from thickening of the boundary layer by large decelerations. Because of these viscous losses and with unloading of the blade, the velocity in the driving face tip is higher than at the trailing face tip. This velocity difference may be expected to result in mixing losses upon diffusion of the air. The experimental distribution of torque and pressure on the blades were compared with an approximate theoretical solution, described in detail in *NACA TN* **2421** [AMR **4**, Rev. 4583]. Test data show good agreement between experimental and theoretical results in the static pressure differences. However, this agreement between theoretical and experimental data is not quite satisfactory as far as the velocity distribution along the blades is concerned. Therefore, the theoretically predicted eddy on the driving face was not found experimentally. H. E. Sheets, USA

2151. **McConaghy, J. W., Centrifugal jet-pump combinations,** *Trans. ASME* **74,** 1, 87–90, Jan. 1952.

Author presents a method of predicting combined centrifugal-jet pump performance from known jet pump performance, centrifugal pump performance, and connecting pipe frictional characteristics. Method is reliable for single liquid system within the accuracy of specifying the pipe friction factors. Brief comments are included on the jet nozzle-to-throat ratio influence on the combined system head and capacity. H. W. Iversen, USA

2152. **Strscheletzky, M., Flow in the transition space of hydraulic turbines** (in German), *Ing.-Arch.* **19,** 4–5, 309–320, 1951.

Author considers a problem of great importance to designers of hydraulic machinery: The calculation of the flow at the runner, as it is established in the transition space between the inlet to the machine and the runner.

A simplified approximate solution of the three-dimensional flow field is presented which dispenses with most of the calculation difficulties inherent in a rigorous three-dimensional treatment. Method is based on representing flow as a set of approximate flows which can be treated by the customary method of conformal transformation. The distribution of vorticity behind the guide names of an axisymmetric three-dimensional inlet is found. Consideration is also given to the calculation of the form of the dead-water space in the inlet, an important factor in the design calculations.

Finally, the theory is applied to flow in the inlet of a Kaplan turbine at two different guide-vane conditions. Agreement with measurements is quite satisfactory.

The method appears to be a relatively short and fairly accurate means of treating the three-dimensional flow in the transition section and should be of considerable interest to designers of turbomachinery. Bruno W. Augenstein, USA

2153. **Spannhake, W., The problem of the flow through turbomachines,** Proc. Midwest. Conf. Fluid Dynamics, 1st Conf., May 1950; J. W. Edwards, Ann Arbor, Mich., 305–309, 1951. $10.

Author describes in general terms methods of improving turbomachine efficiencies by considering blade circulation, boundary layer, and conformal mapping. A relationship is established for torque and head in terms of the circulation outside and inside the runner which is further developed to include effect of number of blades.

Analysis of axial runner as cascade of parallel blades is given. The paper propounds the problems and describes in very general terms various solutions that may be made now and in the future. H. H. Anderson, Scotland

Flow and Flight Test Techniques

(See also Revs. 1985, 2073, 2079, 2111, 2117, 2141, 2143)

2154. **Dumanois, P., The transonic wind tunnel of Modane-Avrieux** (in French), *Mém. Artill. fr.* **25,** 4, 825–848, 1951.

Need for a large high-speed tunnel of 50,000 kw or greater power is indicated. This particular tunnel to be driven by Pelton turbines of 100,000 hp was under construction by the Germans in 1945 at Oetztel in Austria. The difficulties in collecting many parts (2474 tons of them) of tunnel and moving them to France for re-erection at Modane-Avrieux are indicated. Although the help of Professor Peters, designer of the tunnel, was obtained, the design was checked with a one-eighth scale model. This is the one-meter tunnel at Chalais-Mendon which was placed in operation in 1948 and reached a Mach number of one.

The working section of the Modane-Avrieux tunnel is 8 m (26 ft) in diameter, and the developed circuit is 390 m (1280 ft) in length. The nozzle is fairly conventional with a contraction ratio of 9. The 14 m (46 ft) long working section is followed by a diffuser of initial angle of 5 degrees. The four corners contain about 20 vanes each. The two counterrotating impellers or fans of 15 m (49 ft) diameter located between the second and third turns are driven by separate Pelton turbines of 5 m ($16^1/_2$ ft) diameter under a water head of 750 m (2480 ft) and developing a total of 110,000 hp. The sheet-iron skin of the tunnel is 6 to 12 mm thick.

The means for cooling and exchanging the tunnel air, for speed control of the turbines, are described as well as the balances and test carriages. Two interchangeable carriages are provided for aerodynamic studies and one for motor tests.

J. M. Robertson, USA

2155. **Göthert, B., Wind-tunnel corrections at high subsonic speeds, particularly for an enclosed circular tunnel,** *NACA TM* **1300,** 43 pp., Feb. 1952.

Translation from *Forschungsber.* 1216, *Deut. Dersuchsanst. Luftfahrt*, May 1940.

2156. **Erwin, J. R., and Emery, J. C., Effect of tunnel configuration and testing technique on cascade performance,** *NACA Rep.* 1016, 15 pp., 1951.

See AMR **4,** Rev. 1320.

2157. **Hansche, G. E., and Rinehart, J. S., Air drag on cubes at Mach numbers 0.5 to 3.5,** *J. aero. Sci.* **19,** 2, 83–84, Feb. 1952.

The air drag on two sizes of steel cubes, $^1/_4$ and $^3/_8$ in., has been carefully determined in the approximate range from Mach 0.5 to 3.5. In general, the results are consistent and in substantial agreement with similar data obtained by Charters and Thomas on spheres. The present data do not yield an explicit value of the drag coefficient C_DA for a rotating cube, where A is proportional to the area of the cube. An average value for A of $1.50d^2$, where d is the length of one edge of the cube, is assumed and C_D is calculated. The two sizes of cubes give essentially equal values for C_D at corresponding Mach numbers. From authors' summary

2158. **Fearnow, D. O., Investigation of the structural damping of a full-scale airplane wing,** *NACA TN* **2594,** 11 pp., Feb. 1952.

Investigation was conducted by the shock-excitation method wherein the wing was loaded to a predetermined deflection and the load suddenly released. The test specimen vibrated at its fundamental bending frequency of 1.69 cps. Only the first 2 or 3 cycles showed any indication of a higher frequency being superimposed upon the fundamental bending frequency. The damp-

ing was found to increase from about 0.002 of critical at an amplitude of vibration of ± 0.05 in. to approximately 0.006 of critical at an amplitude of ± 5 in. From author's summary

2159. Hilton, W. F., and Fowler, R. G., Photographs of shock wave movement, *Aero. Res. Counc. Lond. Rep. Mem.* 2692, 10 pp., Dec. 1947, published 1952.

Consecutive photographs were taken at millisecond intervals of the flow past a low-drag airfoil at compressibility speeds. At a Mach number 0.1 above the "pressure critical" the shock wave was found to oscillate rapidly but aperiodically, whereas the edge of the associated boundary layer remained quite steady, at least for periods of $^1/_{50}$ sec. At the critical Mach number and just below it, a series of small shock waves was observed, apparently moving against the direction of flow. From authors' summary

Thermodynamics

(See also Rev. 2119)

2160. Hirschfelder, J. O., and Curtiss, C. F., The theory of flame propagation, *J. phys. coll. Chem.* **55,** 6, 774–788, June 1951.

An extension of the theory governing the structure of one-dimensional flames (see authors' earlier paper, AMR **4,** Rev. 414). Topics considered are: Origin and reaction of radicals; approximate and asymptotic solutions of the equations; and the relation between the number of modes required to express the asymptotic behavior and the number of dependent variables that are linearly independent. No numerical applications are made to specific chemical systems. More recent investigations of similar sets of equations (see authors' paper, *Univ. Wisc. nav. Res. Lab. Rep.* CM-690, and reviewer's paper, *Ballist. Res. Lab. Mem. Rep.* no. 563) indicate that the singularities of such problems must be closely investigated before the general equations can be useful.

Bruce L. Hicks, USA

2161. Colburn, A. P., Problems in design and research on condensers of vapours and vapour mixtures, *Instn. mech. Engrs. Proc.* **164,** 4, 448–458, 1951.

The theoretical background is outlined for condensation of pure and mixed vapors in conditions of a film-type condensate. The importance of vapor velocity is shown. Approximations applicable to design are given. Myron Tribus, USA

2162. Focke, R. J., Contribution to the thermodynamics of gas flow, *Cienc. y Técn.* **117,** 594, 229–249, Dec. 1951.

The work presents fundamental relations for the heating of a gas flowing in a duct of constant cross section. The temperature and velocity limitation of this heat addition are determined, and the detrimental effect on the thermal efficiency of heat engines with continuous gas flow is discussed. The possibilities for avoiding these disadvantages are derived for the cases of heating at constant pressure and at constant volume. The extension of the derived relationships to the supersonic region leads to a clear presentation of the normal compression shock which can be described by the simple law of arithmetic mean values. The treatise closes with a discussion of thermodynamic causes which give rise to the origin of compression shock. [Reviewer also suggests as a reference on the same subject, Foa and Rudinger, AMR **2,** Rev. 507.] From author's summary by A. W. Jones, USA

2163. Wall, F. T., and Flory, P. J., Statistical thermodynamics of rubber elasticity, *J. chem. Phys.* **19,** 12, 1435–1439, Dec. 1951.

Authors and, alternatively, James and Guth have contributed to the statistical mechanics of networks consisting of long-chain molecules. These theories arrive at different expressions for the change of entropy associated with the formation of a network from the independent molecules. The discrepancy is due to the fact that different types of networks are envisaged: Whereas James and Guth consider fixed and permanent cross linkages, authors have a network in mind in which the number but not the relative position of cross linkages is constant. Authors emphasize the superiority of their model for representing the properties of rubber; reviewer is not convinced by the reasons given. In addition, authors show that an equation appearing in one of James and Guth's papers [eq. (3) of the present paper] involves an anisotropy of the unstrained network. For this reason, authors claim, the theory of James and Guth is to be rejected.

Reviewer agrees with the necessity of avoiding anisotropy of this kind, but notices that James and Guth use the above equation for the purpose of illustrating a point in a discussion and not in the course of presenting their theory. Minor controversial points are discussed, but without including the extent to which the competing theories actually conform to the properties of those mechanical models which they are to represent.

R. Eisenschitz, England

🕮2164. Khinchin, A. I., translated from Russian by Gamow, G., Mathematical foundations of statistical mechanics, New York, Dover Publ., Inc., 1949, 174 pp. \$1.25.

Author establishes basic formulas of classical statistical mechanics, using limit theorems of probability theory in place of steepest-descent treatment. Results are applied to ideal monatomic gas. Significance of thermodynamic functions is discussed briefly. Quantum statistics is not treated.

Turner Alfrey, Jr., USA

2165. Buff, F. P., The spherical interface. I. Thermodynamics, *J. chem. Phys.* **19,** 12, 1591–1594, Dec. 1951.

Author presents extension of Gibbs thermodynamic theory of spherical interfaces leading to generalized forms for the Kelvin relation and the Gibbs adsorption equation. System discussed is two-phase, single component in the absence of external fields. Modification of Gibbs theory consists in choosing a dividing surface so as to make the superficial mass density vanish rather than the classical surface of tension. Author shows further that, by relaxing the condition of equality of chemical potential between phases while retaining all other thermodynamic relations, an approximate expression may be obtained for the work of formation of a droplet which is not in equilibrium with a large mass of surrounding dilute vapor. The latter formula describes the one-dimensional Eyring surface employed in current nucleation theories, and agrees with the result deduced by Becker and Döring from kinetic considerations. Lawrence M. Grossman, USA

2166. Jenny, E., The utilization of exhaust-gas energy in the supercharging of the four-stroke diesel engine, *Brown Boveri Rev.* **37,** 11, 433–447, Nov. 1950.

Diesel engines offer an attractive field for supercharging with gas-turbine power, because the diesel exhaust temperatures are relatively low and hence the turbine bucket material is not a serious problem, and because excess air can be used for blown-through scavenging without loss of fuel. Author has undertaken a thorough study of the exhaust process and its connection with the scavenging problem from the point of view of unsteady gas dynamics. For energy utilization with shock, rapid opening of the exhaust valves is of greatest importance as well as exhaust-pipe cross section and length, correct grouping of branched pipes, turbine flow section. Test apparatus, working with cold com-

pressed air, consists of four similar cylinders enclosing rotors instead of pistons. For simulating actual pressure variations in cylinder and exhaust pipe, suitable openings are arranged in stator (cylinder) and rotor. The apparatus is flexible, built so that choice of rotor speed, flow cross sections, piping arrangement, and turbine nozzle enable agreement of the dimensionless coefficients with those of an actual engine. Gillis Huss, Sweden

2167. **Jenny, E., Unidimensional transient flow with consideration of friction, heat transfer, and change of section,** *Brown Boveri Rev.* **37,** 11, 447–461, Nov. 1950.

Paper is a comprehensive abstract of author's thesis, Zürich 1949 [AMR **4,** Rev. 2581] giving valuable contributions to general transient gas-dynamic theory and its special application to engine-exhaust problems. Graphical calculations are presented in distance-time and state diagrams. Examples are gas discharge into pipe with open end, with nozzle at extremity and with diffuser, pulsejet tube with heat addition through combustion. Interesting comparisons are made with tests performed in apparatus built for diesel-engine research (see preceding review).
Gillis Huss, Sweden

2168. **Stalder, J. R., Goodwin, G., and Creager, M. O., A comparison of theory and experiment for high-speed free-molecule flow,** *NACA Rep.* 1032, 22 pp., 1951.

See AMR **4,** Rev. 3386.

2169. **Tzeïrov, E. M. Discharge of compressed air from pipes** (in Russian), *Zh. tekh. Fiz.* **21,** 7, 842–852, July 1951.

In first section, author treats theoretically the outflow of gases from cylindrical pipes. In second section, experiments are discussed. The initial pressure was 16 atm and the pressure variation both outside and inside the pipe as a function of time is studied. Author mentions importance of his work for the study of air switches. D. ter Haar, Scotland

Heat and Mass Transfer

(See also Revs. 2053, 2054, 2161)

2170. **Lineïkin, P. S., On the cooling of the surface layer of the sea** (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **80,** 205–208, Sept. 1951.

A method of successive approximation solution for the one-dimensional nonsteady heat-conduction equation is outlined for the case where the effective thermal diffusivity is a product of time-dependent and distance-dependent functions. The procedures and conclusions offer nothing not readily available from existing sources. N. A. Hall, USA

2171. **Farber, E. A., Free convection heat transfer from electrically heated wires,** *J. appl. Phys.* **22,** 12, 1437–1438, Dec. 1951.

Author discusses heat transfer by free convection from electrically heated wires (copper and iron) to distilled water between freezing and boiling, to water boiling at atmospheric conditions, and to air at room temperature. The wire surface temperatures needed for the heat-transfer determination are calculated from the theoretical temperature distribution in the wire. The experimental results show maximum heat-transfer rates of about 2,200,000 Btu/hr ft² near freezing and 450,000 Btu/hr ft² near boiling. For boiling water, as the temperature difference between the wire surface and water is increased, the heat-transfer coefficient first increases reaching a maximum, then decreases reaching a minimum and increases again (because of radiation) until the wire fails by melting. For copper, the maximum film coefficient of 9100 Btu/hr ft² °F is observed at 49 degrees temperature difference and minimum of 136 Btu/hr ft² °F at a temperature difference of 690 degrees F. Corresponding values for iron are 12,300 Btu/hr ft² °F at a 33 degree temperature difference and 142 Btu/hr ft² °F at a 620 degree difference. When air at room temperature surrounds the wire, the heat-transfer coefficient for copper with an oxide film varies from 0 to 30 and for iron from 0 to 26 Btu/hr ft² °F as the surface temperature of the wires is raised until failure occurs. From author's summary

2172. **Kudryashev, L. I., On the connection of generalized integral relations with the hydrodynamic theory of heat transfer and its application to the computation of heat transfer for the conditions of an external problem** (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 11, 1682–1688, Nov. 1951.

Author considers the application of integral relations previously derived [L. I. Kudryashev, AMR **2,** Rev. 393] to the determination of Nusselt number relations for external boundary layers. The results confirm, in particular, the $^1/_3$ power for the Prandtl number and in a conventional manner the Reynolds number dependence from skin-friction expressions.
Newman A. Hall, USA

2173. **Shklover, A. M., Temperature oscillations on the surface of an infinite wall with noncontinuous heat supply** (in Russian), *Zh. tekh. Fiz.* **21,** 11, 1372–1378, Nov. 1951.

Consider heat supply Q constant over a fraction f of the period, and zero over the rest of the period. Author expands Q in Fourier series and obtains temperature of wall in Fourier series. This series is numerically summed for $f = {}^1/_2$ and ${}^3/_4$, and the maximum and minimum values of the periodically varying temperature are given as a function of f. D. ter Haar, Scotland

2174. **Peck, R. E., and Reddie, W. A., Heat transfer coefficients for vapors condensing on horizontal tubes,** *Indust. Engng. Chem.* **43,** 12, 2926–2931, Dec. 1951.

Considerable deviation between experimental results and calculated values has been found when the Nusselt equation is used to calculate film coefficients for a vapor condensing filmwise on a horizontal tube. Two of the assumptions upon which the equation is based (temperature drop across the condensing film is constant, and condensate film is not subject to acceleration forces) are shown to be partially or wholly invalid. Experimental results of authors and sixteen other investigators show that heat transfer around the tube does not account for deviations from the Nusselt equation.

A correlation which corrects a great deal of the discrepancy is developed so that, with the application of the proper dimensionless group given in the correlation, film coefficients can be predicted with greater accuracy.

From author's summary by William J. Anderson, USA

2175. **Burnand, G., The study of the thermal behavior of structures by electrical analogy,** *Brit. J. appl. Phys.* **3,** 2, 50–53, Feb. 1952.

The application of the analogy to the study of heating in dwellings, the maximum size of the units of capacity and resistance in the electrical models, and the methods of connecting up the model components are discussed. Electronic equipment for use with electrical models of houses is described briefly. Comparisons with observed temperature data in a room agreed with electrical model results when the model was modified to account for the presence of temperature gradients and hot airstreams in the actual room. From author's summary

2176. Scadron, M. D., and Warshawsky, I., Experimental determination of time constants and Nusselt numbers for bare-wire thermocouples in high velocity air streams and analytic approximation of conduction and radiation errors, *NACA TN* 2599, 81 pp., Jan. 1952.

Results are presented on the determination of time constants of several bare-wire thermocouples of varying configurations mounted in cross flow to an airstream. The Mach number was varied from 0.1 to 0.9 and the Reynolds number based on wire diameter varied from 250 to 30,000. Results show that the time constant depends primarily on thermocouple wire material, wire diameter, gas pressure, Mach number, and only slightly on gas temperature. The results were correlated in the form

$$N_{Nu} = (0.427 \pm 0.018)\,(N_{Re})^{(0.515\ \pm\ 0.005)}(N_{Pr})^{0.3}$$

with average deviations of 6.9%. The gas properties are evaluated at total temperature. Graphs and nomographs are given for the rapid evaluation of approximate radiation and conduction errors, as well as time constants, for wire diameters between 0.001 and 0.1 inch for commonly used pairs of thermocouple wires.

The effects of dissimilarities in wire material and diameter on the steady-state temperature distribution in a thermocouple and the effect of conduction along the wire on the time constant are analytically treated and results presented.

Y. S. Touloukian, USA

2177. Wexler, A., Evaporation rate of liquid helium. I, *J. appl. Phys.* **22**, 12, 1463–1470, Dec. 1951.

Some of the factors governing the evaporation of liquid helium and the principles of design of efficient storage vessels are reported. Two 9.25 l spherical Dewar type flasks immersed in liquid nitrogen were used; the outer surface of the inner sphere was polished copper in one vessel, and sand-blasted Monel in the other. An electrical heater whose energy input was accurately measured was used to vary the rate of evaporation.

The various types of heat transfer are discussed on the basis of approximate equations. The relation of the emissivity data to the theory of Reuter and Sondheimer is presented.

The hemispherical emissivity of mechanically polished copper at 4.23 K was 0.0069 and 0.0129 for blackbody at 77.1 K and 297.1 K, respectively. The heat of evaporation of helium at 4.228 K was 4.93 cal g^{-1}; this is in good agreement with the value of Dana and Kamerlingh Onnes.

The loss of stored helium was 1% per day, and author states that the theoretical analysis and his data demonstrate the feasibility of constructing storage containers of moderate size that will hold liquid helium for a year or more after the initial filling.

H. E. Robison, USA

2178. Gilman, S. F., Martin, R. J., Konzo, S., Investigation of the pressure characteristics and air distribution in box-type plenums for air conditioning duct systems, *Univ. Ill. Bull.* **48**, 79, Bull. 393, 77 pp., July 1951.

Box plenums having multiple branch ducts are investigated under a variety of test conditions to obtain design data for air-duct systems. Some general conclusions reached are: (1) Total-pressure loss in box plenum is composed of an entrance, a turbulence, and an outlet loss. (2) Design of entrance section of box plenum greatly affects pressure losses and air distribution to ducts. (3) Nearness of fan to box-plenum entrance has little effect on total pressure loss. (4) Lower pressure losses and better over-all performance are obtained when air supply is introduced into box plenum at end instead of at the bottom.

Equations for total loss of box plenums of approximately same dimensions as those studied are stated in paper.

Harrison D. Goodman, USA

Acoustics

2179. Miller, H. B., The discontinuous growth and decay of sound in a one-dimensional room, *J. acoust. Soc. Amer.* **23**, 5, 517–530, Sept. 1951.

Because of recent interest in transient response of rooms, author has published basic work done some years ago on transient response of a helical spring, equivalent to a one-dimensional "room" 58 ft long. By sending audiofrequency compressional waves down the spring from one end and recording oscillograms of the waves picked up at the far end, author found (a) that Eyring's mechanism of image sources did not explain all the effects found, and (b) that a new "two-pipe" mechanism could be envisaged which did explain all the effects. Reviewer believes this work will help in understanding the yet more complicated behavior of sound in three-dimensional rooms.

P. H. Parkin, England

2180. Kurtze, G., Investigation of improvement of the lining of anechoic rooms (in German), *Akust. Beihefte* no. 2, AB104–AB107, 1952.

Paper shows how to improve the sound absorption of absorbing wedges as used in anechoic chambers by means of resonators of the Helmholtz type, arranged behind. The resulting arrangement has the same absorption coefficient with only ²/₃ of the depth of the usual wedges.

From author's summary

2181. Ferrero, M. A., and Sacerdote, G. G., Parameters of sound propagation in granular absorbent materials, *Acustica* **1**, 2, 137–142, 1951.

The acoustic impedance on the surface of samples of 8 granular materials (leaden balls with diameters 3.70, 2.55, 1.95 mm, and sand with diameters 2–2.5, 1–2, 0.5–1, 0.33–0.50, 0.25–0.33 mm), which are placed at the end of an air-filled rigid tube, is measured by a resonance-curve method (methods 6, 7 in L. L. Beranek, "Acoustic measurements," Wiley, New York, 1949, pp. 329–336) in the frequency range 300–1400 cps. From these measurements, the characteristic impedances, attenuation constants, and phase constants are determined, graphically represented, and compared with theoretical data.

Hans L. Oestreicher, USA

2182. Lauer, H., Thermal damping of bubbles of various gases in water (in German), *Akust. Beihefte* no. 1, AB12–AB24, 1951.

Paper reports 1940–1942 experimental studies based on theoretical calculation by Pfriem of heat-conduction component of damping factor for vibration of small bubbles of air, argon, neon, hydrogen, and helium in water. Method employs frequency width of half-power points on resonance curve determined by photomultiplier tube observation of forced oscillation of bubble trapped under parallel wires, small disk, or wire loop in liquid. Resonant frequency of bubble is determined by bubble stiffness (isothermal at low frequencies, adiabatic at high) and radiation mass appropriate to size. Theory is summarized, and experimental difficulties are discussed in much detail. Depth of liquid, distance from walls, type of container, diffusion of gas into water, and type of bubble holder influence results, which show considerable spread but, in general, satisfactorily confirm theory.

Vincent Salmon, USA

2183. Voelz, K., Attenuation of acoustic resonators (in German), *Z. angew. Phys.* **3**, 2, 67–72, 1951.

Paper investigates under several simplifying assumptions the energy loss in an acoustic resonator of arbitrary shape, taking into consideration viscosity (both shear and volume viscosity) and heat conductivity. Both influences are computed separately

assuming, **first, a viscous gas without heat conduction**, and, secondly, **a nonviscous gas with heat conduction**. In the first case (viscosity only), the loss is assumed to be the sum of the energies attenuated in (1) the compression waves and (2) the shear waves along the walls. The first part is calculated in the conventional way from the complex propagation constant. The second part is taken care of by a boundary-layer consideration, assuming plane shear waves.

The second case, the energy loss in a heat-conducting nonviscous gas, is treated analogously.

Hans L. Oestreicher, USA

2184. **Harris, C. M., and Molloy, C. T., The theory of sound absorptive materials,** *J. acoust. Soc. Amer.* **24,** 1, 1–7, Jan. 1952.

Paper, rather short for its purpose, defines and evaluates areas of present knowledge to characterize sound absorptive properties of materials. Early experiment by Sabine introduced absorption coefficient, a number independent of sound field which is not an adequate specification but continues to be a useful quantity in practice. Discrepancies with new experimental data lead to the introduction of normal specific impedance as ratio of complex pressure and complex velocity, which was used by Morse and others in obtaining analytical expressions for sound fields within enclosures and was found valid in many experimental works. Paris derived a relation between normal specific impedance and absorption coefficient from which Sabine and London derived methods for obtaining average absorption coefficient in agreement with its measured values. In some cases it was impossible to use normal impedance boundary condition but instead a more general boundary condition, derived from equation of propagation of sound in homogeneous, isotropic, porous materials which are defined according to Scott by two complex constants: characteristic impedance and propagation constant. Experiments confirm this theory.

Paper concludes with review of recent work on perforated panels, resonator, and suspended absorbers.

Leonardo Villena, Spain

2185. **Noury, J., The velocity of ultrasonics of 960 kc in ethane in the neighborhood of the critical state** (in French), *C. R. Acad. Sci. Paris* **234,** 3, 303–305, Jan. 1952.

In this note are presented the data on the velocity of ultrasonics in ethane, the measuring method being light diffraction by ultrasonics. As for CO_2 and N_2O, one observes a velocity minimum at the critical point; the precision which reached 1.5% is better than before.

From author's summary

2186. **Canac, F., and Gavreau, V., Ultrasonics in air and their applications** (in French), *Acustica* **1,** 1, 2–16, 1951.

Paper describes construction of two types of ultrasonic transducers for use in air up to 75 kcps. One type is an electromagnetic-driven metal cylinder vibrating at resonance; the other type is a magnetostriction device with horn. These are used to plot acoustic scattering from cylinders and various corrugated surfaces. Acoustic fields are also measured in model rooms by an automatic plotting method.

Robert W. Morse, USA

2187. **Sokolov, S. Ya., Ultrasound and its application** (in Russian), **Zh. tekh. Fiz. 21,** 8, 927–936, Aug. 1951.

The title of this article is somewhat misleading, since a general study of applications has not been made. Instead, a number of particular applications are briefly reviewed. Among these are: Flaw detection, the use of velocity measurements to observe the completeness of chemical reactions, the ultrasonic microscope, and the Delye-Sears effect of the diffraction of light by ultrasound.

A more detailed study is presented of the transmission of ultrasound in quartz crystals. In particular, attention is called to the anisotropy of losses in a flawless single crystal, and to the dependence of the energy loss in transmission along grain boundaries on grain size, orientation, and acoustic wave length.

R. T. Beyer, USA

2188. **Barthel, R., and Nolle, A. W., A precise recording ultrasonic interferometer and its application to dispersion tests in liquids,** *J. acoust. Soc. Amer.* **24,** 1, 8–15, Jan. 1952.

The device is an interferometer of the two-crystal type. It operates with low acoustical power and with advancing wave in order to prevent the heating effect. The wave is very plane. These safeguards assure a true measurement of the sound velocity. The acoustical signal is sent out by a stationary crystal at one end of the tank and received at the other end by a moving crystal. The received frequency is slightly different from the original because of the Doppler effect. Both frequencies are beat against another. The beat frequency is amplified, limited to give a square wave with the beat periodicity, and differentiated to give two very sharp pips per period. The negative pip is suppressed, leaving a series of sharp pips with exactly the same periodicity as the beat signal. The output activates a stylus marking a line on a rotating drum at every pip whenever the receiving crystal has moved a distance of one wave length in the liquid. Rotation of the recording drum and translation of the crystal are linked together so that the wave length may be derived from the known mechanical constants of the system. However, corresponding to the sound velocity which is estimated beforehand, the linkage must be a continuously variable lever until lines fall side by side, starting ideally from a straight line parallel to the axis of the drum. Normally, the interferometer operates at 5 mc but also operates at 15 mc or at 25 mc. While at 5 mc the stylus traces one line at one revolution of the drum, it does it three times at 15 mc or five times at 25 mc. A dispersion would be detected as a slight change of the starting line between different frequencies.

Velocity of sound in thin solid samples may be measured by recording the phase shift when the sample is suddenly removed from the sound beam during the operation. This phase shift causes a discontinuity in the record trace. The interferometer has a precision of the dispersion measurement of about 0.8 part in 10,000; the precision of the absolute determination of sound velocity in liquids is 0.2 m/sec; in solid samples, however, it is less remarkable. The precision of temperature is ± 0.01 C. One operation runs only 15 sec. In 4 pure liquids and 10 solutions, sound velocity and dispersion are examined at several values of temperature. The results are critically compared with those of earlier authors. Particularly, sound velocity of water was found to be lower than known hitherto and in no case a dispersion could be found, i.e., dispersion was either unknowable or within the limits of precision of the measurement.

Peter-Paul Heusinger, Germany

2189. **Stevenson, A. F., General theory of electromagnetic horns,** *J. appl. Phys.* **22,** 12, 1447–1460, Dec. 1951.

Exact equations for the propagation of electromagnetic waves in a perfectly conducting horn of arbitrary shape are given. They take the form of an infinite set of simultaneous ordinary linear differential equations, and can be interpreted as the equations of propagation of a system of coupled E- and H-waves. If the coupling is neglected, we need only consider a single differential equation for each E- and H-wave, which can be solved approximately by the W.K.B. method. This approximate solution brings out the distinction between "transmission regions" and

"attenuation regions" of the horn, as found by Barrow and Chu for the sectoral horn.

It is shown that the error due to neglect of coupling is, in general, of the order of the square of the flare angle as far as the variation of the field along the horn is concerned, but is of the first order in the flare angle as regards the variation of the field over the cross section. The coupling cannot, however, be neglected between modes of propagation which have the same cut-off frequency for all cross sections. The propagation characteristics of several special shapes of horn are discussed in detail.

From author's summary

2190. **Benioff, H., Ewing, M., and Press, F., Sound waves in the atmosphere generated by a small earthquake,** *Proc. nat. Acad. Sci. Wash.* **37,** 9, 600–603, Sept. 1951.

Theoretical results of a study of coupling of Rayleigh waves to atmospheric compressional waves are applied to an observed atmospheric wave train generated by an earthquake. The air wave train radiated by the Rayleigh waves travels with the speed of sound in air, includes all frequencies from zero to $KH \simeq 2$, where the group velocity drops abruptly. The computed times of arrival of the first and last air waves are in good agreement with observations.

H. L. Kuo, USA

2191. **Rust, H. H., and Bailitis, E., Critical considerations of the linear magnetostrictive ultrasonic generation by means of oscillators of dumbbell type** (in German), *Akust. Beihefte* no. 2, AB89–AB90, 1952.

It is shown that the transverse magnetostrictive effect reduces the resulting amplitude of longitudinal magnetostrictive oscillators of the dumbbell type. With transducers of conventional size using nickel as ferromagnetic material, the out-of-resonance amplitude is less than half that value which can be obtained from the magnetostriction curve, the loss of amplitude due to remanence being taken into account.

From authors' summary

2192. **Esche, R., Investigations of ultrasonic absorption in animal tissues and plastics** (in German), *Akust. Beihefte* no. 2, AB71–AB74, 1952.

A method is described for measuring the absorption of ultrasonic waves in animal tissues and in plastics in the frequency range 300–600 kcps. Measurements prove the absorption coefficient to increase linearly with frequency. They agree well with results obtained by other authors at higher frequencies.

From author's summary

2193. **Mintz, F., and Tyzzer, F. G., A loudness chart for octave-band data on complex sounds,** *J. acoust. Soc. Amer.* **24,** 1, 80–82, Jan. 1952.

Two important contributions are made to the equivalent-tone method for calculating the loudness of continuous spectra noise [Beranek, Marshall, Cudworth, and Peterson, AMR **5,** Rev. 265]. Sound pressure levels are converted to loudness values for use of contours of equal pressure level in octave bands. This graphical method of finding an equivalent pure tone at the geometric mean frequency of each band makes use of the Churcher-King equal loudness contours and the Fletcher-Munson loudness curve. Because the procedure is based on the use of octave bands, instead of 300 or 600-mel bands suggested by Beranek, it is possible to employ octave-band filter sets which are readily available commercially. Comparison of loudnesses calculated for a number of practical noise sources with three different sets of bands (5 bands-600 mels, 10 bands-300 mels, 8 octave bands) indicates that octave bands can be used to give satisfactory results for engineering purposes.

E. G. Fischer, USA

2194. **Laird, D. T., and Cohen, H., Directionality pattern for acoustic radiation from a source on a rigid cylinder,** *J. acoust. Soc. Amer.* **24,** 1, 46–49, Jan. 1952.

Paper investigates radiation patterns from sources located on an infinite cylinder. Using method of stationary phase, far zone field is derived for separable source functions of azimuth and axial dimensions. General theory is then applied to uniform rectangular source. Horizontal and vertical polar patterns of sound pressure amplitude and phase were computed on IBM card-programmed calculator for cylinder circumference-to-wave-length ratio of 14.

Osman K. Mawardi, USA

Ballistics, Detonics (Explosions)

(See also Rev. 2157)

2195. **Garnier, M., Remarkable points on a trajectory. V. Curvilinear trajectories. Point of an element of given value. VI. Contact with the safety curve** (in French), *Mém. Artill. fr.* 25, 4, 747–804, 1951.

Chapter V [for preceding chapters see AMR **4,** Rev. 4009; 5, Revs. 275, 1896] contains the computation of elements of the trajectory, if any element has a given value. Following cases are treated: Points where the angle between the tangent to the trajectory and the (curved) earth is given (e.g., the summit); points with given height (the point of fall belongs to these points); points with given abscissa, distance (inclined range), angle of position, or time of flight. As the safety curve envelops all trajectories with the same initial velocity but different angles of departure, chapter VI explains the variations of several elements, if the angle of departure changes for a small amount (10′).

H. Schardin, Germany

2196. **Leimanis, E., The application of infinitesimal transformations to the integration of differential equations of exterior ballistics by quadratures,** Proc. Second Canad. Math. Cong., Vancouver, 1949, 206–217, Univ. Toronto Press, 1951. $6.

The projectile is considered a material point acted on by only two forces, gravity and drag, the latter being in the tangent to the trajectory. The function in the hodograph equation is $f(v)$ and the density of the air is $\delta(y)$. The Lie theory is applied to show that in the following cases the variables can be separated: $f = v$, $\delta = (1 + by)^n$ for

$$n = -2, -\tfrac{1}{2}, 0, 1;\ f = v,\ \delta = e^{ky},\ f = v^2,\ \delta = (1 + ky)^{-1};$$
$$f^{-1} = v^2 \exp(av^2/2g);\ \delta = \exp(-ay).$$

It is said that these include all known and some new integrable cases.

Courtesy of Mathematical Reviews J. M. Thomas, USA

2197. **Crocco, G. A., Temperature barrier in geodetic missiles** (in Italian), *Atti. Accad. naz. Lincei R. C. Cl. Sci. Fis. Mat. Nat.* (8) **10,** 2, 97–103, Feb. 1951.

This is an introductory study to the title subject. Article deals with considerations related to the path of a body submitted to the earth's gravitational central field. Coriolis acceleration and aerodynamic forces are neglected. No thrust is considered.

Trajectory is an ellipse with one focus at the center of the earth, provided the speed is less than escape velocity. The range appears to be a function of the launching angle and initial velocity. The particular launching angle minimizing initial velocity for a *given* range is calculated.

Results agree with those previously obtained by Ivey, Bowen, and Oborny [AMR, **1,** Rev. 565]. Angelo Miele, Argentina

2198. **Crocco, G. A., Temperature barrier in geodetic missiles. II. Dynamic of a missile with propulsive jet** (in Italian), *Atti Acad. naz. Lincei R. C. Cl. Fis. Mat. Nat.* (8) **11**, 1/2, 3–10, July/Aug. 1951.

This study deals with the partial pattern flown by a missile in the neighborhood of the earth after its launching.

External forces are: The thrust of a propulsive jet, gravity, and the aerodynamic forces. Acceleration of gravity is supposed to be constant. Author carries out a number of observations concerning the physical nature of the problem. Integration of the trajectory equations must be performed step-by-step.

Angelo Miele, Argentina

2199. **Dupuis, J., Influence of initial yaw on the range and duration of the trajectory** (in French), *Mém. Artill. fr.* **24**, 3, 555–575, 1950.

The effect, on range and time of flight of a shell, of the drag caused by an initial yaw is considered equivalent to a decrease in initial velocity proportional to maximum yaw squared. Comparison with limited experimental data is satisfactory. Applications to dispersion, etc., are discussed.

A. S. Galbraith, USA

2200. **Price, E. W., Steady-state one-dimensional flow in rocket motors,** *J. appl. Phys.* **23**, 1, 142–146, Jan. 1952.

The problem of the internal ballistics of a solid-propellant rocket is treated in one dimension. The usual gas-flow equations are written down and the rate-of-burning law is taken as proportional to a power of the pressure and to $1 + Ku$, where u is the stream velocity. The mass flow equation is written in a form suitable for numerical solution in terms of a parameter β. Various physical quantities, e.g., pressure distributions, are graphed against β (which is proportional to the distance along the charge). The equilibrium conditions are evaluated for a rocket with JPM ballistite in terms of β and the channel-to-throat ratio J. Solutions, given graphically, cover a wide range of nozzle charge size ratios.

R. C. Knight, England

2201. **Bebb, A. H., Under-water explosion measurements from small charges at short ranges,** *Phil. Trans. roy. Soc. Lond.* (A) **244**, 879, 153–175, Dec. 1951.

Author describes experiments made to obtain pressure time records of underwater-explosion waves. The explosives used were TNT and PE in mass between 2 oz and $1^1/_2$ lb. The measurements were taken with a tourmalin gage of $^1/_2$ or $^1/_4$-in. diam. Author attempts to measure the shock-wave pressure up to the surface of the explosive, which is new, compared with other publications using the same method.

Reviewer has compared the results with his own measurements taken by an optical method which does not disturb the phenomenon. He found a sufficient agreement, considering that, the explosives were not exactly the same. Reviewer means that, for the region nearest the explosive, optical methods are to be preferred.

Besides the peak pressure, author gives results for the mean shock-wave velocity, the time constant for the decrease of the pressure in the shock wave, the impulse of the shock wave, and the energy flux in the shock wave and in the afterflow. The afterflow energy-flux density is of the same order as the primary shock-wave energy-flux density, especially near the charge. Author compares his results with the existent theories of underwater explosions.

H. Schardin, Germany

Soil Mechanics, Seepage

(See also Revs. 1977, 2006, 2020, 2021, 2222, 2235)

2202. **Meyerhof, G. G., The ultimate bearing capacity of foundations,** *Géotechnique Lond.* **2**, 4, 301–332, Dec. 1951.

Starting from the conditions of failure of weightless material with internal friction and cohesion, author improves the method of estimating ultimate bearing capacity of foundations. Previous theories take into account the effect of foundation depth considering the respective vertical pressure, due to the overburden, as a surcharge and use the same sliding surface as by surface loading (logarithmic spiral between sliding planes of active and passive Rankine state). Author, making use of the method suggested by Prof. J. Jáky, correctly traces the sliding surface over the horizontal plane of foundation in a logarithmic spiral toward the side wall of the footing and for shallow foundations toward the free surface, respectively. After Terzaghi's method, the formula giving the ultimate bearing capacity is composed of three terms: They take into consideration the cohesion and friction, as well as depth and breadth of the footing. Bearing-capacity factors are computed to determine the named terms. Formulas of general bearing-capacity factors are given for two cases: (a) There is no friction on the equivalent free surface, i.e., along the line connecting corner point of footing with end point of sliding surface; and (b) friction is fully mobilized on this plane. It is stated that numerically there is little difference between these cases. Bearing-capacity factors are presented in graphs as function of angle of internal friction with the inclination of equivalent free surface as a parameter. It is to be noted that, in the case of cohesion material, resistance increases slightly only with foundation depth; as to the skin friction, the opposite effect occurs.

In the second part, results of laboratory and field loading tests are summarized. In clay, the observed ultimate bearing capacity is in good agreement with theoretical results and also in sand in case of shallow foundation. For deep foundation, the actual bearing capacity is somewhat less than the estimated one; this fact is ascribed to the occurrence of local shear failure. In all cases, the method of installing the foundation (buried or driven) has a marked effect.

Reviewer believes this to be an important contribution for estimating bearing-capacity factors, and hopes that author will continue his investigation to include some effects so far neglected (frictional forces in the base, eccentric loadings, rigidity of foundation slab, etc.).

Árpád Kézdi, Hungary

2203. **Ruppeneĭt, K. V., Verification of the strength of mine roofs** (in Russian), *Izv. Akad. Nauk. SSSR Otd. tekh. nauk* no. 4, 579–592, Apr. 1951.

Plane strain methods of the theory of elasticity are applied to the determination of stresses in mine roofs of a span L, whereby the roof proper is formed by a uniform layer of thickness h. The following three cases are considered: (1) $h \geqslant L$; (2) $h < L$; (3) $h \leqslant {}^1/_4\, L$. Depending on the strength of the overlying strata, the roof is treated as a one-layered or as a two-layered system. Diagrams and numerical examples illustrate the application of the formulas derived.

Gregory P. Tschebotarioff, USA

2204. **Kersten, M. S., and Cox, A. E., The effect of temperature on the bearing value of frozen soils,** *Nat. Res. Counc. Highway Res. Bd. Bull.* no. 40, 32–38, 1951.

Effect of variations in stability with temperature, density, moisture content, and soil texture was investigated to indicate significance for structural uses of frozen soils in permafrost regions. Four soils of different known densities and moisture con-

tents were placed in a cylinder, frozen, and tested; a thermocouple was embedded for temperature measurements. California bearing-ratio type of tests were made with bearing value arbitrarily taken as unit load for 0.1-in. penetration of steel rod. Similar tests were made on ice.

In general, results showed increases in bearing values with decreases in temperature, with increases in density, and with increases in moisture content. Bearing values of sand were much higher than for ice; values for sandy loam were slightly higher than for ice. Values for silt loam were slightly lower and for clay were much lower than for ice.

Plots showed that, for small decreases in temperature below 32 F, the bearing values for soils increased rather rapidly; as temperatures were further lowered toward −5 F, the rate of increase of these values decreased, giving a curve relationship. Plots of bearing values vs. temperature for ice were represented by a straight line.

Reviewer believes that results for ice should have been represented by a curve similar to that for soils on the basis of (1) experimental points and (2) such results as those for compressibility of ice presented by N. E. Dorsey, "Properties of ordinary water-substance," New York, Reinhold Publishing Corp., 1940.

George J. Tauxe, USA

2205. dos Santos, M. P. P., Prediction of liquid limit and plasticity index of mixtures of soils (in Portuguese), *Rev. Ordem Engrs.* **8,** 76, 181–188, Apr. 1950.

Author gives the following formulas: $I.P. = (A_1a_1I_1 + A_2a_2I_2)/(A_1a_1 + A_2a_2)$, $L.L. = (A_1a_1L_1 + A_2a_2L_2)/(A_1a_1 + A_2a_2)$, where $I.P.$, $L.L.$ are plasticity index and liquid limit, respectively; I_1, I_2 plasticity indexes of the mixed soils; L_1, L_2 liquid limits of the mixed soils; A_1, A_2 weight percentages of each of the mixed soils; a_1, a_2 granulometric magnitudes of the soils.

Experimental evidence for the validity of the formulas within the limits of practice is presented.

From author's summary

2206. dos Santos, M. P. P. Prediction of the shrinkage limit, linear shrinkage, and the equivalent humidity of soil mixtures (in Portuguese), *Rev. Ordem Engrs.* **9,** 94, 452–456, Oct. 1951.

Previous work of the author (see preceding review) has shown how the introduction of a new soils constant provides a simple tool for the provision of the liquid limit, plastic limit, and plasticity index of mixtures of soils, through the formula $K = (A_1a_1k_1 + A_2a_2k_2)/(A_1a_1 + A_2a_2)$, where K is characteristic of the mixture (liquid limit, plasticity index, etc.); k_1 and k_2 corresponding characteristics of the soils to be mixed; A_1 and A_2 percentage by weight of each component soils; a_1 and a_2 the new constants, related to the grain size curve of the soils to be mixed, through the expression: $a = \Sigma y/100\ n$, where y is the ordinate (percentage passing) of the grain-size curve corresponding to nos. 200, 100, 52, 25, and 7 B.S. sieves; and n the number of such ordinates being taken.

Present paper extends these results to the case of the shrinkage limit and linear shrinkage of mixtures of soils, and comes to the rather startling conclusion that the field moisture equivalent does not obey the same law of variation, but presents a linear relationship.

Figs. 1 and 2 show results of two typical tests; figs. 3, 4, and 5 give the statistical frequency of discrepancies between the theoretical and the actual values for about 100 mixtures of very different types of soils.

From author's summary

2207. Schultze, E., Strict solutions of earth pressure problems and their significance in practice (in German), *Baupl. Bautech.* **4,** 3, 69–77, Mar. 1950.

Paper was intended originally as a review of Jáky ("Sur la stabilité des masses de terre complétement plastiques"), but author broadened its scope so as to treat, in a general form, the basic concepts of such theoretical investigations and their usefulness to practical applications.

Theoretical treatment of behavior of any kind of material must be based necessarily on very simplified assumptions. Thus, problems of soil mechanics are reduced to ones in which the soil may be considered to be in a state of plastic or a state of elastic equilibrium, depending on whether the deformations involved are big enough to produce surfaces of sliding or not.

In the case of elastic equilibrium, results of the theory of elasticity may be applied. If the soil is in a state of plastic equilibrium, theoretical treatment is vastly complicated by the fact that the principle of superposition is no longer valid. The results of the theory of plasticity are of great practical interest, especially since recent investigations have shown that soils with horizontal shear diagram have to be considered much more frequently than formerly.

From this point of view, author gives a qualitative analysis of a considerable number of important problems in soil mechanics.

Eckhardt Rathgeb, Argentina

2208. Huizinga, T. K., Application of results of deep penetration tests to foundation piles, Build. Res. Congr., Sept. 1951, Div. I, part 3, 173–179.

Paper compares the results of pile load tests with bearing capacities estimated by Delft cone-penetration method.

Delft apparatus consists essentially of a cone of 10 cm² base area, attached to the end of a rod fitting freely inside of a tube having same outside diameter as cone. Test method consists in forcing first cone and then tube into ground in short increments, until full desired depth of investigation is reached. Data thus obtained, after applying the proper factor of safety, indicate the point resistance and side friction to be expected for driven piles. Load and pull tests on piles are recorded, indicating degree of accuracy of method for conditions in Holland. Cost of investigations by the penetration method is stated as being much less than cost of pile load tests, and accuracy of determining required pile lengths is considered better than that of other methods.

Reviewer believes that the method represents an excellent contribution in the field of preliminary soil surveys, where conditions are similar to those in Holland. However, he warns against too free general use of method in areas where a correlation has not been made between pile load tests and penetration records.

Frederick J. Converse, USA

2209. Nasberg, V. M., The problem of filtration from a source in nonsaturated soil (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 9, 1415–1429, Sept. 1951.

Liquid streams under influence of gravity from a source at $x = 0$, $r = 0$ located within an infinite homogeneous soil, and in steady-state flow, fill a region approximating a paraboloid of revolution opening downward ($x > 0$). Soil outside this region is unaffected. An older work [V. V. Ivakin, "On filtration from a source in nonsaturated soil," title source no. 9, 1947] assumes the flow determined by the source at $x = 0$, $r = 0$ with parallel downward flow superimposed. Author points out that with Ivakin's solution, points on the surface of separation do not satisfy the desired conditions for the head, $h = -x$. Author improves this aspect of the solution considerably by adding a sink of properly chosen strength at $x = -b$.

R. E. Gaskell, USA

2210. **Baumann, P., Ground-water movement controlled through spreading,** *Proc. Amer. Soc. civ. Engrs.* **7.**, Separ. no. 86, 38 pp., **Aug.** 1951.

Paper presents thorough analysis of seepage problems which arise in connection with the technique of enrichment of ground-water supply by the method of "spreading" widely used by the Los Angeles County Flood Control District. Treatment of two-dimensional problems is based on Boussinesq's simplified equation for unsteady seepage flow. Simple cases of axially symmetrical three-dimensional flow, due to circular spreading area or to a recharge well, are treated by approximate methods. Results of small scale experiments indicate good consistency with computations.
Oscar Hoffman, USA

2211. **Uchida, S., On nonsteady percolation with a free boundary** (in Japanese), *J. Japan. Soc. civ. Engrs.*, **37**, 2, 10–14, Feb. 1952.

Nonsteady motion of the ground water through the dike of rectangular cross section is investigated theoretically and experimentally. The form of the free surface of the ground water is calculated by graphical method and is approximately

$$y_f/H = 1 - (3\alpha/8)^{3/4}\,\{(x_f/H)/(\kappa t/H)^{1/2}\}^{3/2}$$

where x_f, y_f are coordinates of the free boundary surface, the origin being taken at the upstream toe of the dike; H is the y coordinate of the upstream water surface; α is porosity; κ is permeability; and t is time. Experiments were performed with lead shot and highly viscous liquid, aiming to exclude the effects of the capillarity. Agreement of the results with the theory is quite satisfactory. The experiments on trapezoidal cross section are also reported.
T. Mogami, Japan

2212. **Florin, V. A., Earth compaction and seepage with variable porosity taking into account the effect of molecularly bound water** (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 11, 1625–1649, Nov. 1951.

Before seepage flow is stabilized even under constant boundary conditions, an unsteady state is present which is characterized by gradual transfer of load from the water to the soil skeleton. As the soil compacts from the increased load, its void ratio and permeability diminish and the seepage pattern is modified. The effect of this factor, ignored in the usual theories of seepage and consolidation, is considered in the new, so-called theory of compaction, the credit for which is claimed exclusively for the Soviet scientists.

In a number of experiments involving downward flow through vertical tubular permeameters provided with piezometers, a curved shape of hydraulic grade line was observed under conditions of steady flow in all soils, indicating a smaller permeability at the bottom, where the intergranular pressure was the greatest, than at the top. A mathematical derivation of the differential equation of the theory of compaction is then given, claimed to be in agreement with the above experiments.

Apart from the variable value of the coefficient of permeability, the compaction equation seems to be similar to the equation of consolidation of Terzaghi except that it is derived on the basis of a modified Darcy equation in which the hydraulic gradient is replaced by the difference of it and a quantity called the initial gradient. The flow thus cannot occur unless the hydraulic gradient is greater than the initial gradient, which modifies or even changes completely the conditions of seepage and consolidation.

Thus, the values of the coefficients of permeability derived from the permeability tests on the basis of the usual Darcy equation always come out too small and variable, depending on the head and thickness used. The amounts of settlement computed by the ordinary theory of consolidation are always too great, because preservation of initial gradient leaves regions far removed from the drainage layer, partially or even fully unconsolidated.

The author discusses at length the final boundary conditions peculiar to a number of one-dimensional problems, and outlines principal steps in their solution, without, however, going into the mathematical technique.

The subject matter of the paper is interesting and thought-provoking, and its implications are very important. The presentation, however, is clumsy and unconvincing as well as pretentious.
Alexander Hrennikoff, Canada

2213. **Payne, P. C. J., and Fountaine, E. R., A field method of measuring the shear strength of soils,** *J. Soil Sci.* **3**, 1, 136–144, 1952.

A field method of determining shear strength is described in which a cylinder of soil is sheared in torsion and a moment vs. angle-of-twist (proportional to strain) curve obtained. From this, the shear-stress vs. angle-of-twist can be obtained mathematically. Where only the maximum shear strength is required, a simplified method is given. A number of results using the apparatus on widely different soil types is given, together with a comparison of some of these with results from the standard shear box.

From authors' summary by Woodland C. Shockley, USA

2214. **Casagrande, L., Electro-osmotic stabilization of soils,** *J. Boston Soc. civ. Engrs.* **39**, 1, 51–83, Jan. 1952.

Author reviews the principles of electroosmosis as applied to soil stabilization and describes four successful field applications. The power consumption varied from 0.4 kwh per cu yd of excavated soil for large excavations to 2 kwh for smaller (900 cu yd) excavations. An explanation is offered for shrinkage and cracking observed in clay under electroosmotic treatment. This is based on the theory that gas bubbles inside the soil form a boundary which interrupts the electroosmotic flow and, in fact, causes a reverse flow down the center of the capillary. In the region of the gas bubble, there is a tension which produces cracks in the soil, and between gas bubbles pore-water tensions produce compression and shrinkage.
Eben Vey, USA

2215. **Kern, L. R., Displacement mechanism in multi-well systems,** *J. Petr. Technol.* (*Petr. Trans.* **195**) **4**, 2, 39–46, Feb. 1952.

Paper deals with oil-field developments in porous strata by displacement methods (water or gas injection), and follows Buckley and Everett's study ["Mechanism of fluid displacement in sands," *Trans. AIME*, **146**, 107, 1942]. Present paper is more general, including also the case of producing wells located between liquid-liquid interface and place of injection. Treatment is analytical and results in formulas for calculating saturation distribution, cumulative oil production, etc., under various conditions. Calculations are simplified by introducing empirical formula for the saturation at interface, and are found to be valid at least for the range between breakthrough and abandonment. Illustrative example shows calculation procedure in detail, but it is not clear if data used apply to an actual field development.
P. Wilh. Werner, Sweden

2216. **Sokolov, Yu. D., On the flow of ground water into a dra nage ditch of trapezoidal section** (in Russian), *Prikl. Mat. Mekh.* **15**, 6, 683–688, Nov./Dec. 1951.

This is an extension and correction of an earlier treatment of the same problem [Polubarinova-Kochina and Fal'kovich, AMR **2**, Rev. 280; translated in "Advances in applied mechanics," AMR **4**, Rev. 4331].

Courtesy of Mathematical Reviews H. P. Thielman, USA

2217. Polubarinova-Kochina, P. Ya., On the dynamics of ground water with sprinkling (in Russian), *Prikl. Mat. Mekh.* **15**, 6, 649–654, Nov./Dec. 1951.

In this paper there is considered the outflow of ground water from a mound of ground water produced by the uniform sprinkling of a bounded region of a pervious layer of soil supported by an impervious layer. The results consist in a graphic description of the known explicit solution $u(x, t)$ of the differential equation $u_t = a^2u_{xx} + f(x, t)$ under various boundary conditions.

Courtesy of Mathematical Reviews H. P. Thielman, USA

2218. Galin, L. A., Some problems of unsteady motion of ground water (in Russian), *Prikl. Mat. Mekh.* **15**, 6, 655–678, Nov./Dec. 1951.

The problems considered deal with the outflow of water from mounds of ground water in layers of infinite or finite depths. The solution gives the height of the free surface of the water, and the velocity potentials in terms of complex integrals of the Cauchy type.

Courtesy of Mathematical Reviews H. P. Thielman, USA

2219. Kochina, N. N., The plane problem of the outflow of a mound of ground water in a layer of infinite depth (in Russian), *Prikl. Mat. Mekh.* **15**, 6, 679–682, Nov./Dec. 1951.

This is another treatment of one of the problems considered by L. A. Galin (see preceding review). The method used here is an adaptation of one used by N. E. Kochin ["Sobranie sochinenii" (Collected works), **2**, pp. 277–304, Moscow-Leningrad, 1949] in the treatment of an analogous problem in the theory of waves on the surface of an incompressible fluid.

Courtesy of Mathematical Reviews H. P. Thielman, USA

2220. Raedschelders, H. M., Frictional resistance in soil (in Dutch), *Tech. Wet. Tijdschr.* **21**, 1, 2; 8–11, 25–31; Jan., Feb. 1952.

From different sources in literature, a general review is given of what is known about the shearing resistance of soils, cohesive as well as noncohesive.

The determination of the shearing resistance of soils by means of shearing tests, triaxial tests, and cell tests is described. A short description of some soil-mechanics calculations is given.

F. C. de Nie, Holland

2221. Meyerhof, G. G., The tilting of a large tank on soft clay, *Proc. South Wales Inst. Engrs.* **67**, 2, 53–71, Feb. 1951.

An investigation is described concerning the settlement of a tank 124 ft in diameter founded on a layer of soft clay of variable thickness up to 10 ft maximum, overlying compact sand and gravel. Settlements increased with the thickness of the clay, and the differential movements caused tilting and fracture of the raft foundation. An analysis of the movements in terms of plastic flow, "elastic settlement," and settlement due to consolidation shows reasonably close agreement with the observations. Attention should be drawn to the plastic analysis, several features of which are novel.

A. W. Skempton, England

Micromeritics

2222. Hawksley, P. G. W., The effect of concentration on the settling of suspensions and flow through porous media, "Some aspects of fluid flow," London, Edward Arnold & Co., 114–135, 1951. 50s.

Arguments are presented to justify an expression for the settling velocity of suspensions applicable to high concentrations. It is supposed that the existence of a uniform settling rate implies an equilibrium distribution such that no relative motion or rotation of the particles occurs. Under these conditions the influence of the other particles on the settling rate of a particular article is regarded as equivalent to an apparent increase of local viscosity, and their motion in a closed vessel leads to a mean upward velocity and an effective increase of density of the fluid.

For spheres, these effects are accounted for by factors applied to the Stokes velocity; for irregularly shaped particles two further factors are introduced, one being a shape factor for a single particle, and the other taking account of the modification of apparent viscosity due to shape.

Experimental evidence in support of theory is found in published data; for dispersed spheres there is good agreement from 7–50% volume concentration, and for nonspherical particles the shape factors deduced from the formula agree with typical values for similar materials. Comparison with the theory extended to flocculated suspensions is also satisfactory so far as the evidence will allow, and there is good agreement in case of settling of nonrigid particles (blood cells).

The formula for flocculated suspensions is also applied to the problem of viscous flow through beds of particles and is found to agree quantitatively with previous theoretical and experimental relations.

Paper is accompanied by a comprehensive list of references.

G. H. Lean, England

Geophysics, Meteorology, Oceanography

(See also Revs. 1966, 2015, 2190)

2223. Hsu, E. H., A general equation of horizontal mass divergence in the atmosphere, *J. Meteor.* **8**, 6, 395–397, Dec. 1951.

A mathematical expression is presented which gives the horizontal mass divergence in terms of the curvature of the streamlines and trajectories of the air, its tangential acceleration, and the vertical gradient of velocity. The expression is mathematically exact and takes account of the curvature of the earth; friction is neglected. Author states that of the 14 terms in the expression only 5 are always negligible. This intricate piece of analysis is useful because only incomplete expressions have been given by earlier authors.

J. S. Sawyer, England

2224. Saito, N., On the turbulence in the lower troposphere, *J. meteor. Soc. Japan* **29**, 11, 378–383, Nov. 1951.

Middle-scale turbulence, defined as turbulence with scale between that of wind-tunnel turbulence and that of very large scale atmospheric turbulence, is studied by analyzing meteorological phenomena in the lower troposphere. The phenomena are oscillations of the wind with a period of 1 to 10 minutes, the diffusion of volcanic ashes, and the eddies in thunderstorms. Data, including those from other sources, are given for values of the diffusion coefficient k (cm²/sec) between 10^3 and 10^{11} and for values of the scale L (cm) between 10^3 and 10^9. These data follow Richardson's law: $k = 0.2L^{4/3}$. The ratio of the mean velocity to the root mean square of the turbulence component is independent of scale for data covering a large range of scale. (Reviewer's note: There appear to be several printing errors in the exponents of the tabular data.)

Neal Tetervin, USA

2225. Riley, G. A., Parameters of turbulence in the sea, *J. mar. Res.* **10**, 3, 247–256, Dec. 1951.

Author develops a statistically valid empirical law for coefficients of eddy diffusivity (vertical and horizontal) as function of stream velocity, scale, and stability. Although there is no ex-

planation of the law, it may perhaps be suggestive for future theoretical development and may prove to be a useful way of expressing the exceedingly diverse coefficients.

Henry Stommel, USA

2226. **Conover, J. H., A new combination anemometer, wind vane and wind direction recorder for Mt. Washington observatory,** *Bull. Amer. meteor. Soc.* **32,** 10, 386–390, Dec. 1951.

A new recording anemometer and wind vane is designed to operate successfully under severe icing conditions. Wind speed is measured by the difference between Pitot and static tube pressure; the Pitot tube is held into the wind by a vane which also serves to measure the wind direction. Position of the vane is transmitted to a remotely located recorder by means of "Selsyn" motors. Pitot-static head, vane, and supports are compactly designed and electrically heated to discourage ice formation.

Franklin I. Badgley, USA

2227. **Berson, F. A., On the factors controlling the instability of long waves in zonal currents,** *Ark. Geofys.* **1,** 187–236, 1951.

The motion of the atmosphere is considered on the basis of the linearized equations of motion and of continuity together with the assumption that an additional physical variable, called the polytropic characteristic, has a zero derivative "following the motion." The rotation of the earth is neglected. The wave solutions of the equations are studied and criteria of instability obtained. As examples of the many results obtained may be cited (1) that horizontally uniform baroclinic zonal currents in middle and high latitudes are dynamically unstable; (2) that the properties of waves of maximum growth rate explain features of rapidly deepening disturbances in the westerlies; (3) small-scale mechanical turbulence is chiefly controlled by the Richardson number, while large-scale turbulence is controlled by this number together with the dimensions of incipient eddies.

Courtesy of Mathematical Reviews G. C. McVittie, England

2228. **Scorer, R. S., Gravity waves in the atmosphere,** *Arch. Meteorol. Geophys. Bioklimatol* (A) **4,** 176–193, 1951.

Gravity waves of period short enough for the earth's rotation to be neglected are considered. The waves are propagated through air moving horizontally, whose velocity and static stability vary with height. The problem is reduced to a differential equation for the vertical velocity as a function of height. The boundary conditions are zero vertical velocity at the ground and finiteness of kinetic energy. Examples are worked out for airstreams in which the stability and velocity are given functions of the height: it is proved that very different results are obtained according as these functions are changed. Unstable waves are also considered and it is argued that they can occur only in very special circumstances and cannot lead to a phenomenon in which one wave length predominates.

Courtesy of Mathematical Reviews G. C. McVittie, England

Lubrication; Bearings; Wear

(See also Rev. 2097)

2229. **Burwell, J. T., and Strang, C. D., On the empirical law of adhesive wear,** *J. appl. Phys.* **23,** 1, 18–28, Jan. 1952.

Authors start from the premises that wear increases in a general way with distance of travel or time of running, but generally not in a linear fashion, and that wear generally decreases with increasing hardness of the rubbing surfaces.

A simple flywheel apparatus is described with a rubbing pin of several shapes, cylindrical, conical, and spherical. For a number of different low loads, the volume of abraded material is linear with distance of travel, provided that a steady condition of operation was reached before any data were taken. The highest load corresponded to about $^2/_3$ the yield strength of the rider material. Experiments in a higher stress region, where the rate of wear increases very rapidly with stress to relatively high values, indicate that the sharp increase in slope of the $h/L{\cdot}P$ vs. pressure curve (h is average depth of material removed, L distance of travel, P average normal stress over the nominal contact area) cannot be a result of breakdown of the liquid lubricant film at some critical stress. The bearing of the results on the running-in of machine parts is discussed.

R. Schnurmann, England

2230. **Blok, H., Viscosity-temperature-pressure relationships, their correlation and significance for lubrication,** 3rd World Petr. Congr., The Hague, 1951. Proc., Sect. 7, 304–319.

Paper is a review and extension of correlations of pressure-temperature-viscosity characteristics of some lubricants and an evaluation of the effect of these relations on the performance of oil-film bearings. First section reviews existing studies of empirical P-T-μ relations. Such empirical relations are desirable in order that viscosity at any pressure and temperature may be predicted from atmospheric pressure characteristics. Second section deals with the relation of these variables in a "similar group" of Pennsylvania oils. Blok postulates that for a "similar group," isotherms plotted on μ-P coordinates are similar. This implies that μ at atmospheric pressure alone determines viscosity at any pressure.

Most important phase is considered in third section which deals with the influence of pressure viscosity upon oil-film bearings. Principal conclusion reached is that only moderate increase in load-carrying capacity might be expected.

Reviewer questions value of pressure-viscosity data obtained in falling ball-type viscosimeters in study of oil-film lubrication. The steady pressure, low shear-rate conditions in the viscosimeter are unlike the pressure-time relations found in bearings.

Blok's conclusion that friction may be greatly affected but the load-carrying capacity only slightly influenced by pressure viscosity does not appear to be consistent. Both the film pressure and the shear stress are dependent upon the local viscosity, and it would therefore be expected that both of these factors would be affected only in the area of high film pressure. His analysis does not include the pronounced influence of temperature increase in the oil film. Raising the viscosity locally would inevitably result in a more rapid local rate of heat generation, resulting in a rise in temperature of the film in that area tending to reduce μ.

Joseph B. Bidwell, USA

Marine Engineering Problems

(See also Revs. 2022, 2134)

2231. **van Manen, J. D., Results of prolonged tests on a Schnitger propeller** (in Dutch), *Schip en Werf* **19,** 4, 69–72, Feb. 1952.

A Schnitger propeller is a propeller with a ring of airfoil section and about half propeller diameter, rotating with propeller. Tests in Dutch model tank with propellers for coaster and trawler showed from 3 to 8% less efficiency than ordinary propeller. Best results (3.3% less efficiency) were obtained with angles of incidence of ring airfoil section 8.5 and 10.8 degrees.

Georg Vedeler, Norway

2232. van Manen, J. D., Influence of the irregularity of the speed field on the design of ship screws (in Dutch), *Ingenieur* **64,** 2, 0.7–0.11, Jan. 1952.

In 1919 Betz and Prandtl showed, by means of circulation theory, that in a homogeneous velocity field a propeller with minimum energy loss could be obtained if ideal efficiency was kept constant over propeller radius. For single-screw ships where entrance velocity for inner part of propeller blade is small, this gave very small load on inner part. Author defines a wake propeller as a propeller which is adjusted in such a way to wake that efficiency of system ship plus propeller is maximum. He concludes that to obtain this it may be of value to increase load on inner part of blades. Helmbold and Lerbs have succeeded in carrying through theoretical calculation for minimum loss of wake propellers, but it is too complicated to be of practical use. Instead, author makes use of Wageningen standard series for open water propellers, although he admits that calculation according to circulation theory may give better results. From standard series he obtains connection between dimensionless velocity coefficient and thrust or power coefficients at diameter of maximum efficiency. This connection is used for making design diagrams by means of circulation theory for wake propellers. No such diagram has, however, been given in paper. Method may give somewhat better results than unadapted standard series. But the limits of such series due to choice of blade sections, pitch, etc., cannot be totally overcome. For diameters less than optimum, method is also complicated.

Georg Vedeler, Norway

2233. Todd, F. H., Skin friction resistance and the effects of surface roughness, *Shipbuilder* **58,** 519, 754–761, Dec. 1951.

See AMR **4,** Rev. 3105.

2234. Schoenherr, K. E., Progress in the computation of the frictional resistance of ships, "Hydrodynamics in modern technol." Hydrod. Lab., Mass. Inst. Technol., 101–110, 1951.

Paper presents a survey of Prandtl's and von Kármán's earlier computations of the turbulent frictional resistance of plates and of the author's analysis of experimental data which led to the determination of the constants in von Kármán's equation. The resulting "Schoenherr line," presenting the frictional resistance coefficient plotted over the Reynolds number (established 1932), is now widely used as the pertinent standard reference curve in naval architecture. Author notes the close agreement of his results with the corresponding ones by H. Schlichting (1934) and Schulz-Grunow (1940).

Georg P. Weinblum, Germany

2235. Christensen, G. N., and Williams, E. J., Diffusion in wood, *Austral. J. appl. Sci.* **2,** 4, 415–453, Dec. 1951.

2236. Archer, S., Contribution to improved accuracy in the calculation and measurement of torsional vibration stresses in marine propeller shafting, *Instn. mech. Engrs. Proc.* **164,** 3, 351–356, 1951.

A further development of the method of determining propeller damping originally proposed by F. M. Lewis ["Torsional vibration in the Diesel engine," *Trans. Soc. nav. Arch. mar. Engrs.* **33,** 109–140, 1925]. Also a repetition of AMR **4,** Rev. **500,** note on phase shift in propeller shaft.

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Biomechanics

2237. Peterson, L. C., and Bogert, B. P., A dynamical theory of the cochlea, *J. acoust. Soc. Amer.* **22,** 3, 369–381, May 1950.

Authors study an idealized model with experimentally found geometrical and mechanical constants as functions of one space-coordinate x $(0 \leqslant x \leqslant l)$. Two channels with equal cross sections $S_0(x)$ represent scala vestibula and scala tympani. These are separated by a membrane (basilar membrane) of width $b(x)$, stiffness $k(x)$, and a constant mass density m, including the mass of the cochlear duct; $S_0(x)$ and $b(x)$ are linear functions, $k(x)$ is exponential. Model is driven at the oval window $(x = 0)$; the round window is terminated by a membrane; driving forces and responses are small; basilar membrane is an assemblage of independent pistons; dissipation is disregarded.

Results: Equations for planar compression waves are set up together with boundary conditions at $x = 0$ (round window) and $x = l$ (helicotrema). Velocity of propagation of waves and pressures is calculated and plotted against x. Broad agreement is found between these data and experiments of Békésy. An equivalent circuit for the cochlea model is analyzed.

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